

Search for Conical Emission in $A + A$ collisions

Claude A. Pruneau
Wayne State University



- Search Method
- Cumulant Definition
- No Such A Thing As ZYAM
- Sensitivity to Conical Emission
- STAR results
- Summary

The Robustness of a Discovery is
only as strong as the assumptions
used to achieve this discovery.

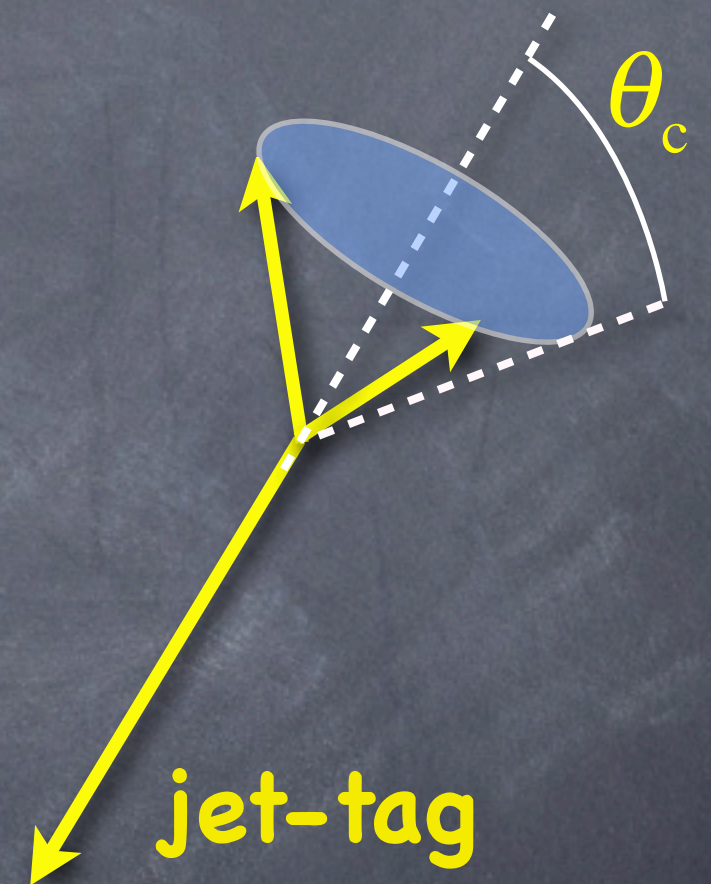
Unfortunately ZYAM is NOT robust.

No Such A Thing As ZYAM

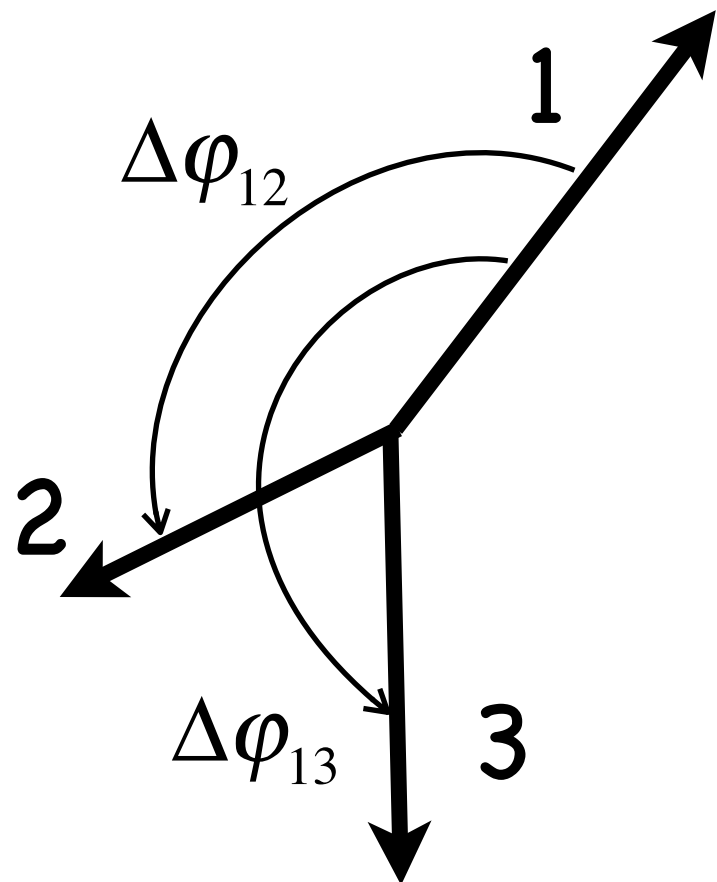
- Two/Three Particle Decays
- Anisotropic Flow Effects
- Momentum/Energy Conservation
- Radial Flow Effects
- $p+p$ is never really gone:
 - Surface Bias
 - Core vs corona correlations
 - Which dominates?
 - Corona correlations may be modified by flow, scattering, absorption, etc

Cone Search Methods

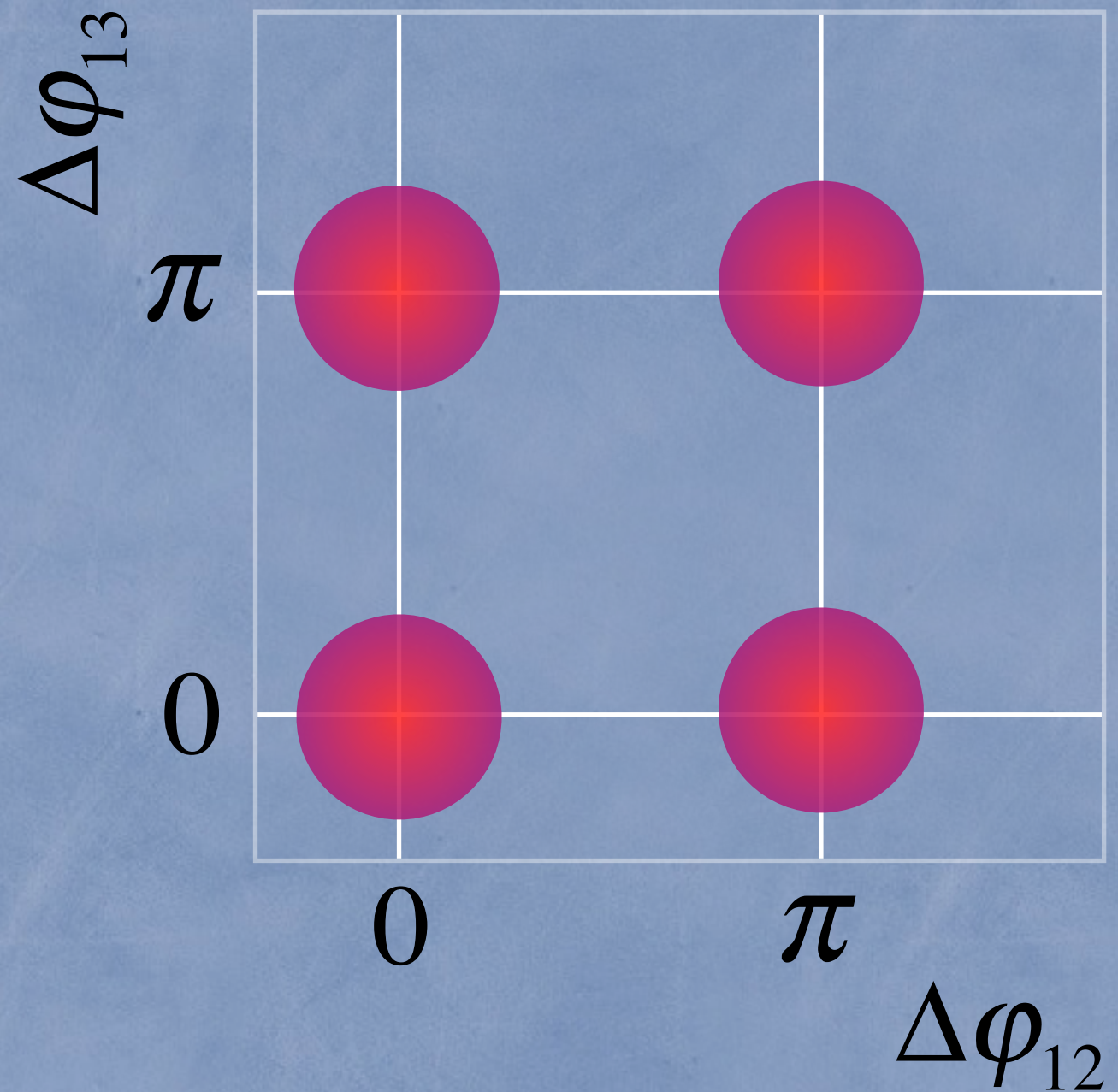
- Use **one high-pt particle** to identify a jet candidate,
 - Trigger particle or **jet-tag**
- Use **two "low-pt" away-side particles** to identify a cone.
 - Transverse Plane
 - Azimuthal Coordinates



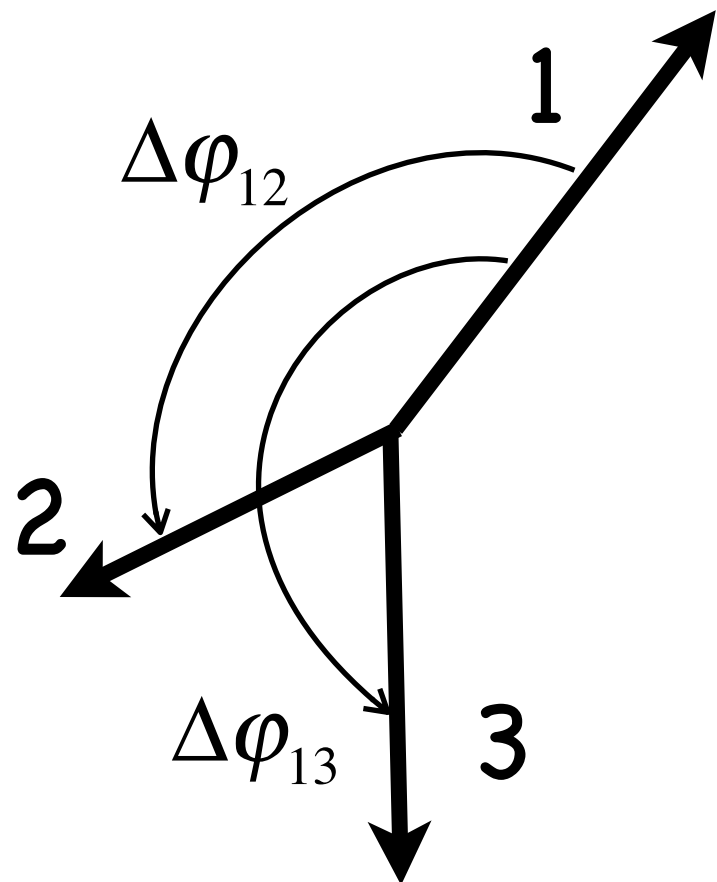
Conical Emission Signature



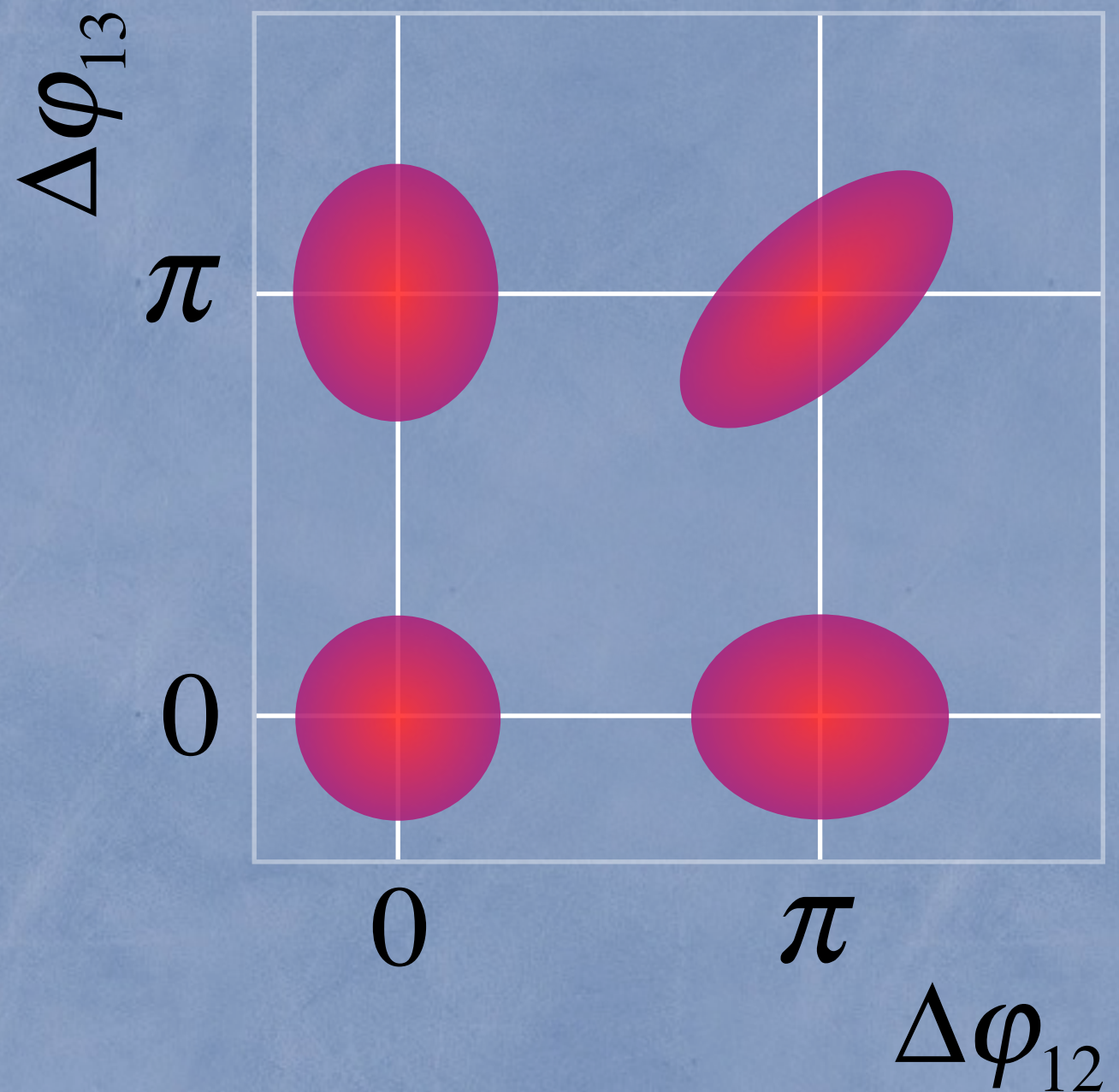
1: Jet Tag
2,3: Associates (Cone)



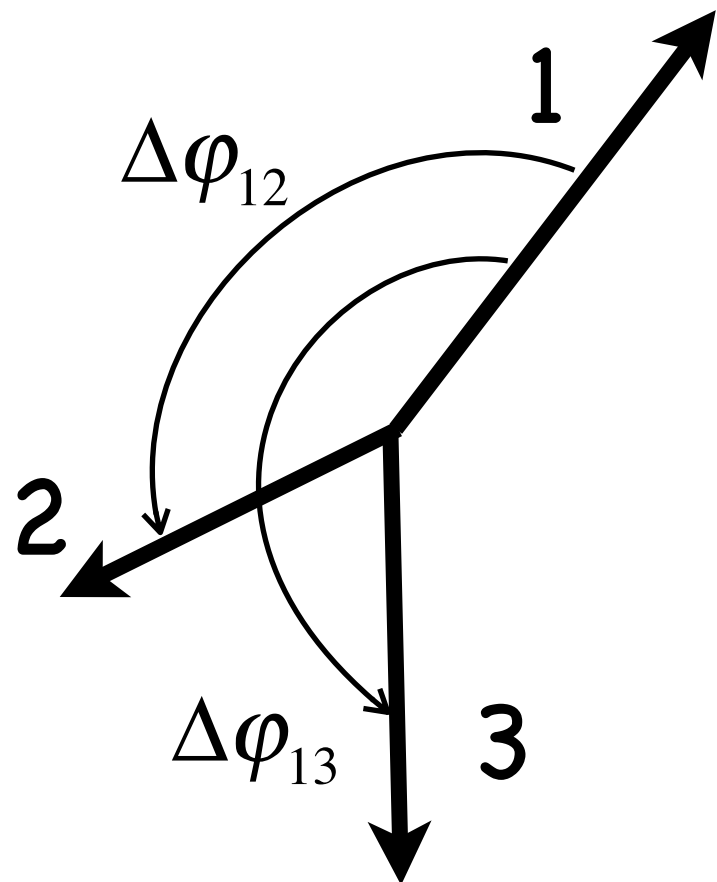
Conical Emission Signature



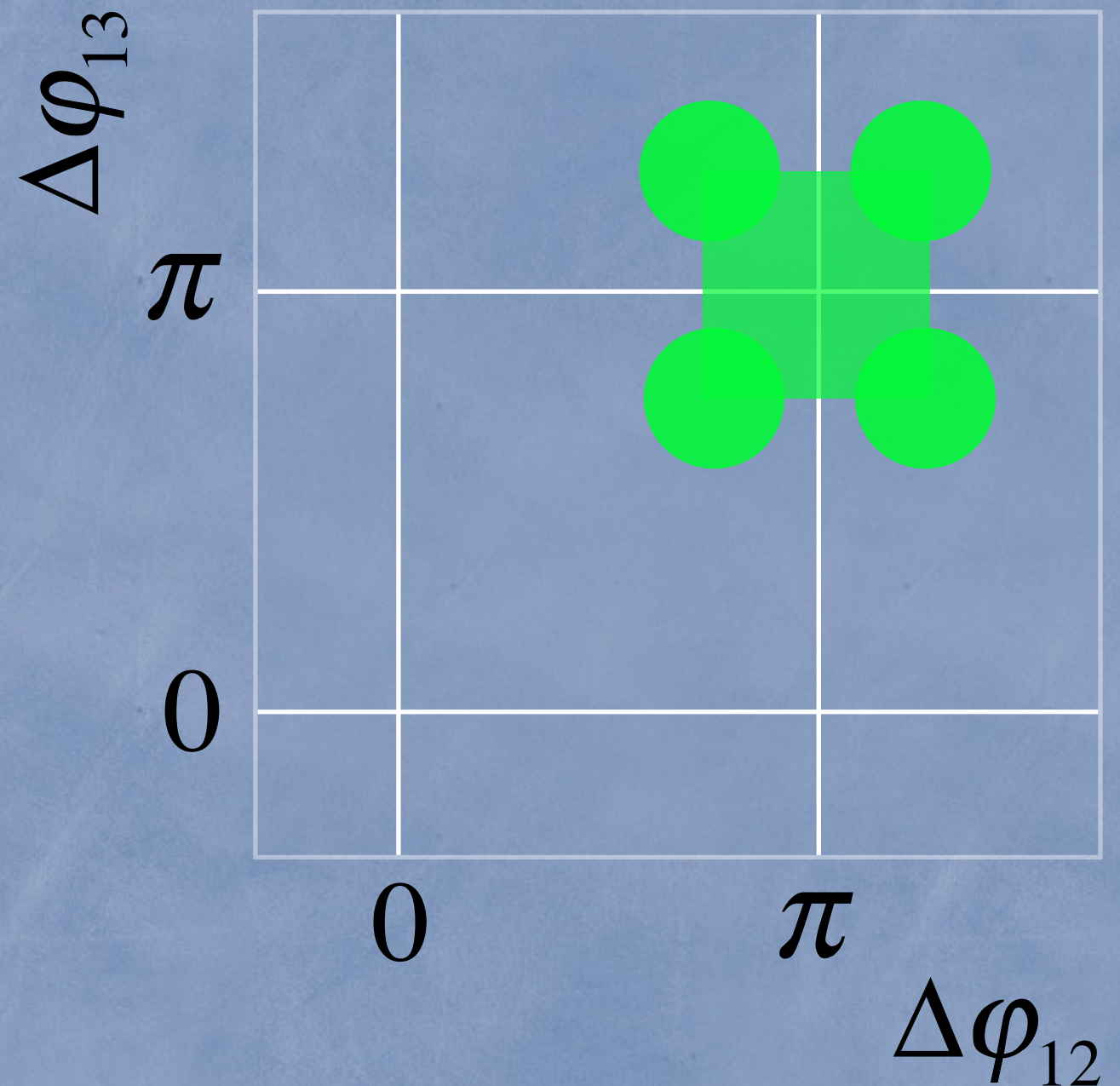
1: Jet Tag
2,3: Associates (Cone)



Conical Emission Signature



1: Jet Tag
2,3: Associates (Cone)



3-Particle Correlation Measurements: Searching for Conical Emission

- What is measured: **3-particle density**
- What is needed: **3-particle correlation**
 - Measure cumulants
 - Or use model to subtract “background” (ZYAM type approaches)
- Issues **(aka Why I do not like ZYAM)**
 - Remove combinatorial backgrounds
 - Collective Flow: Azimuthal & Radial
 - Momentum/Energy Conservation
 - Quantum Number Conservation
 - Resonance Decays
 - Measurement Robustness

Cumulant: Definition

Notation: Measured 1-, 2-, and 3-Particle Densities

$$\rho_1(\varphi_i) \equiv \frac{d^2 N}{d\varphi_i} \quad \rho_2(\varphi_i, \varphi_j) \equiv \frac{d^2 N}{d\varphi_i d\varphi_j} \quad \rho_3(\varphi_i, \varphi_j, \varphi_k) \equiv \frac{d^3 N}{d\varphi_i d\varphi_j d\varphi_k}$$

2- and 3-Cumulants

$$\begin{aligned} C_2(\Delta\varphi_{12}) &= (2\pi)^{-1} \int d\varphi_1 d\varphi_2 \hat{\rho}_2(\varphi_1, \varphi_2) \delta(\varphi_1 - \varphi_2 - \Delta\varphi_{12}) \\ &= \rho_2(\Delta\varphi_{12}) - \rho_1(1)\rho_1(2) \end{aligned}$$

$$\begin{aligned} C_3(\Delta\varphi_{12}, \Delta\varphi_{13}) &= \rho_3(\Delta\varphi_{12}, \Delta\varphi_{13}) - \rho_2(\Delta\varphi_{12})\rho_1(3) - \rho_2(\Delta\varphi_{13})\rho_1(2) \\ &\quad - \rho_2(\Delta\varphi_{13} - \Delta\varphi_{12})\rho_1(1) + 2\rho_1(1)\rho_1(2)\rho_1(3) \end{aligned}$$

Probability Cumulant

$$P_3(\Delta\varphi_{12}, \Delta\varphi_{13}) = \frac{\rho_3(\Delta\varphi_{12}, \Delta\varphi_{13})}{\langle N_1 N_2 N_3 \rangle} - \frac{\rho_2(\Delta\varphi_{12})\rho_1(3)}{\langle N_1 N_2 \rangle \langle N_3 \rangle} - \frac{\rho_2(\Delta\varphi_{13})\rho_1(2)}{\langle N_1 N_3 \rangle \langle N_2 \rangle} \\ - \frac{\rho_2(\Delta\varphi_{13} - \Delta\varphi_{12})\rho_1(1)}{\langle N_2 N_3 \rangle \langle N_1 \rangle} + 2 \frac{\rho_1(1)\rho_1(2)\rho_1(3)}{\langle N_1 \rangle \langle N_2 \rangle \langle N_3 \rangle}$$

Key Cumulant Property

- In a system involving multiple INDEPENDENT sources (processes), the Cumulant of the whole system is equal to the sum of the cumulants of each of the sources.
- Enables to factorize the different effects contributing to the measured cumulant
 - Exception: System wide correlations e.g. Momentum conservation.

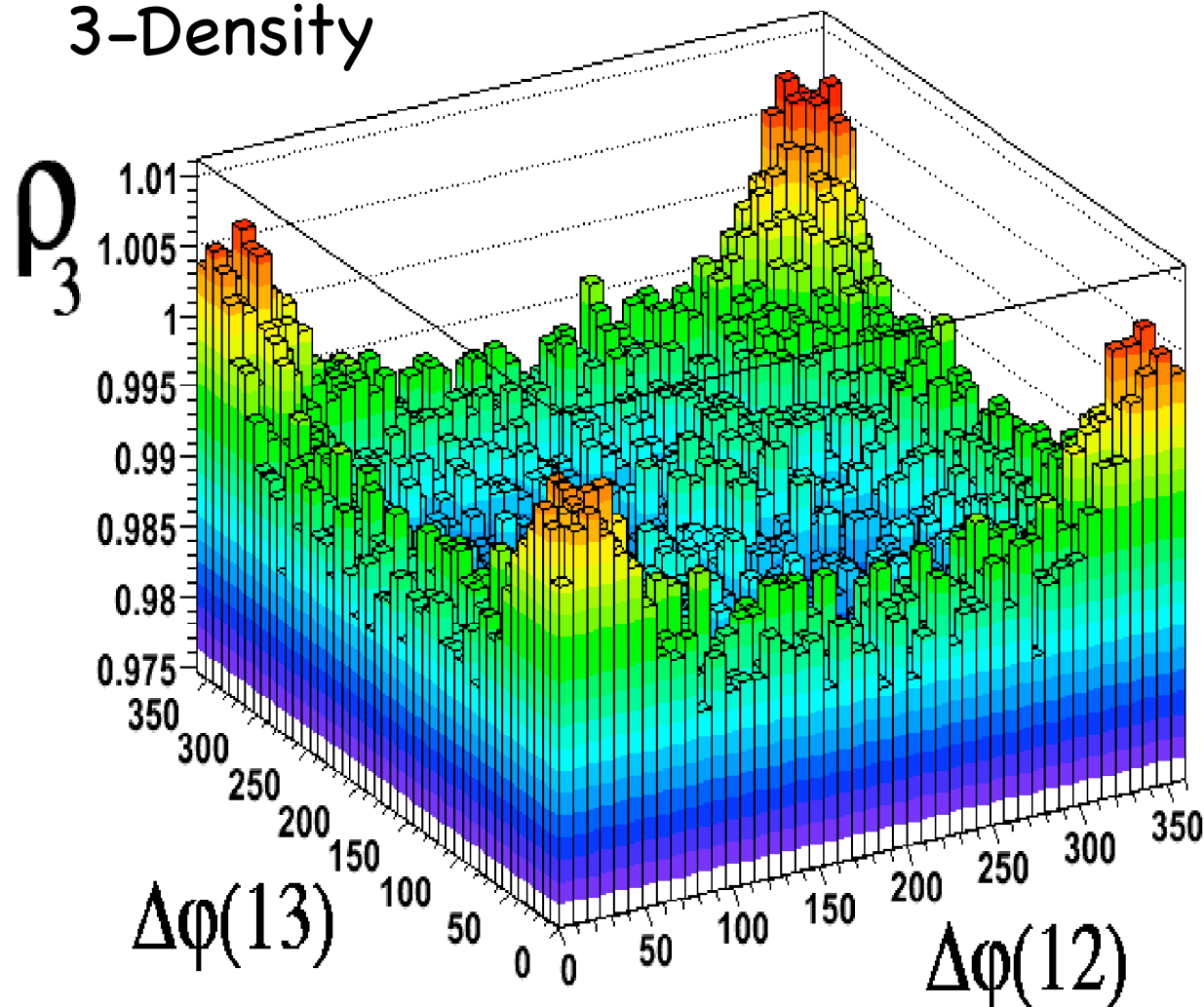
3-Cumulant:

Suppress 2-particle correlations

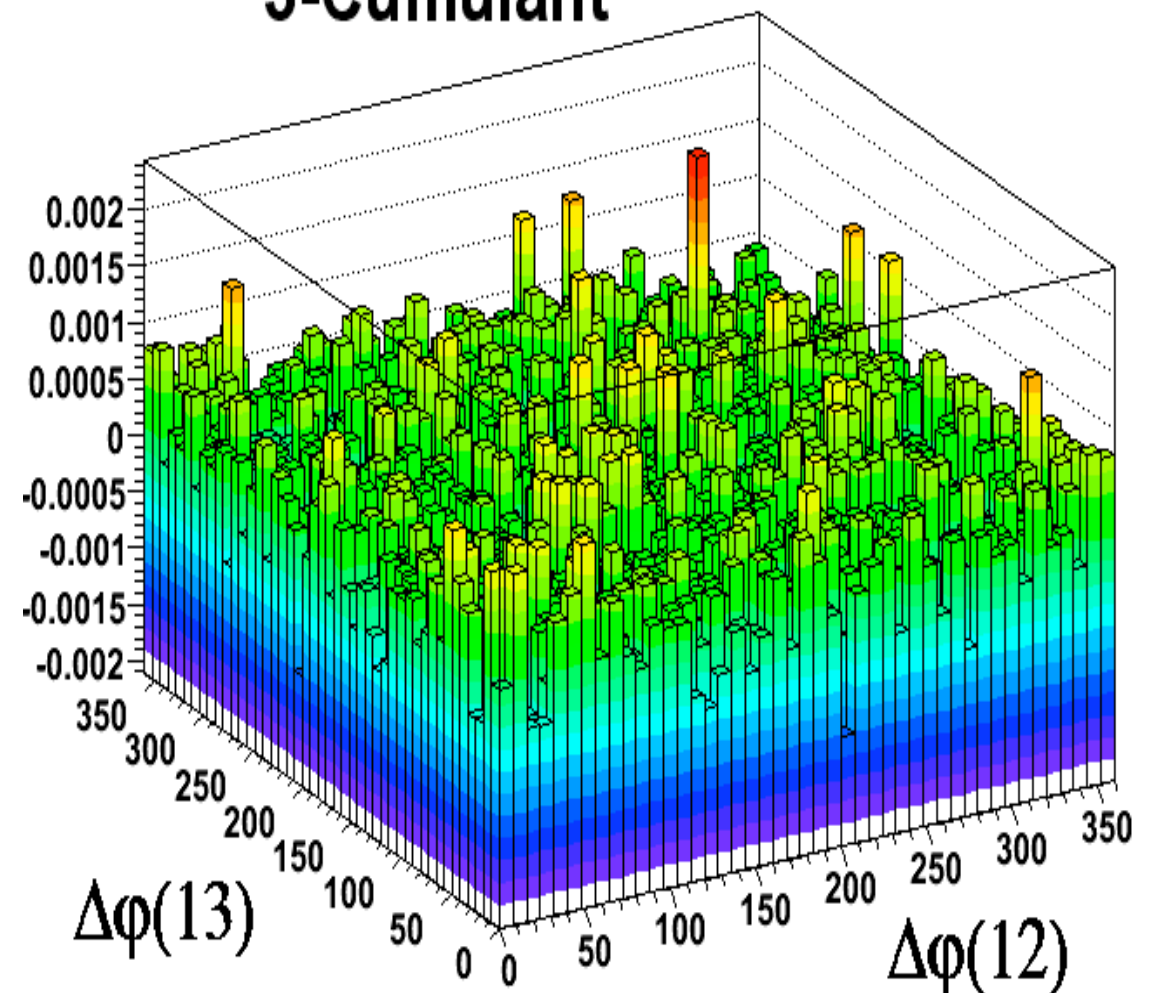
Example: Two-particle decay simulation:

Thermal Thermal Rho Spectrum $\rho \rightarrow \pi^+ + \pi^-$

3-Density



3-Cumulant



Anisotropic Flow

- All particles are correlated to the reaction plane direction.
- Implies: All particles are correlated.
- Correlations exist at all orders...

$$P_F(\varphi_i | \psi) = 1 + 2 \sum_m v_m(i) \cos(m(\varphi_i - \psi))$$

$$C_{2,Flow}(\Delta\varphi_{ij}) = (2\pi)^{-2} \langle N_i N_j \rangle \left(1 - d_{ij} + 2 \sum_m v_m(i) v_m(j) \cos(m\Delta\varphi_{ij}) \right)$$

$$C_{3,Flow}(\Delta\varphi_{ij}, \Delta\varphi_{ik}) = (2\pi)^{-3} \langle N_i N_j N_k \rangle \left\{ \begin{array}{l} \Phi_3(\Delta\varphi_{ij}, \Delta\varphi_{ik}) \\ + (1 - f_{ijk}) \Phi_2(\Delta\varphi_{ij}) + (1 - f_{ikj}) \Phi_2(\Delta\varphi_{ik}) + (1 - f_{jki}) \Phi_2(\Delta\varphi_{jk}) \\ + 1 - f_{ijk} - f_{ikj} - f_{jki} + 2g_{ijk} \end{array} \right\}$$

$$\Phi_2(\Delta\varphi_{ij}) = 2 \sum_m v_p(i) v_m(j) \cos(p\varphi_i - m\varphi_j - n\varphi_k)$$

$$d_{ij} = \frac{\langle N_i \rangle \langle N_j \rangle}{\langle N_i N_j \rangle}; \quad f_{ij} = \frac{\langle N_i N_j \rangle \langle N_k \rangle}{\langle N_i N_j N_k \rangle}; \quad g_{ij} = \frac{\langle N_i \rangle \langle N_j \rangle \langle N_k \rangle}{\langle N_i N_j N_k \rangle}$$

$$\Phi_3(\Delta\varphi_{ij}, \Delta\varphi_{ik}) = 2 \sum_{p,m,n} v_p(i) v_m(j) v_n(k) \left[\begin{array}{l} \delta_{p,m+n} \cos(p\varphi_i - m\varphi_j - n\varphi_k) \\ + \delta_{m,p+n} \cos(-p\varphi_i + m\varphi_j - n\varphi_k) \\ + \delta_{n,m+k} \cos(-p\varphi_i - m\varphi_j + n\varphi_k) \end{array} \right]$$

Anisotropic Flow

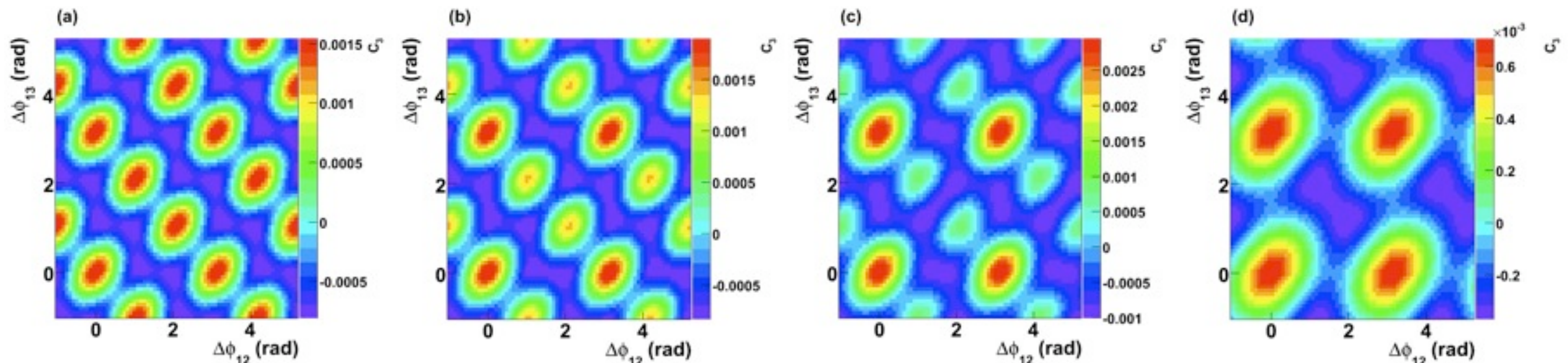
C.P., In print PRC, [arXiv:0810.1461](https://arxiv.org/abs/0810.1461)

$$\Phi_2(\Delta\varphi_{ij}) = 2 \sum_m v_p(i) v_m(j) \cos(p\varphi_i - m\varphi_j - n\varphi_k)$$

$$d_{ij} = \frac{\langle N_i \rangle \langle N_j \rangle}{\langle N_i N_j \rangle}; \quad f_{ij} = \frac{\langle N_i N_j \rangle \langle N_k \rangle}{\langle N_i N_j N_k \rangle}; \quad g_{ij} = \frac{\langle N_i \rangle \langle N_j \rangle \langle N_k \rangle}{\langle N_i N_j N_k \rangle}$$

$$\Phi_3(\Delta\varphi_{ij}, \Delta\varphi_{ik}) = 2 \sum_{p,m,n} v_p(i) v_m(j) v_n(k) \left[\begin{aligned} &\delta_{p,m+n} \cos(p\varphi_i - m\varphi_j - n\varphi_k) \\ &+ \delta_{m,p+n} \cos(-p\varphi_i + m\varphi_j - n\varphi_k) \\ &+ \delta_{n,m+k} \cos(-p\varphi_i - m\varphi_j + n\varphi_k) \end{aligned} \right]$$

3-Cumulant



$$d_{ij} = f_{ij} = g_{ij} = 1$$

$$v_4 > 0$$

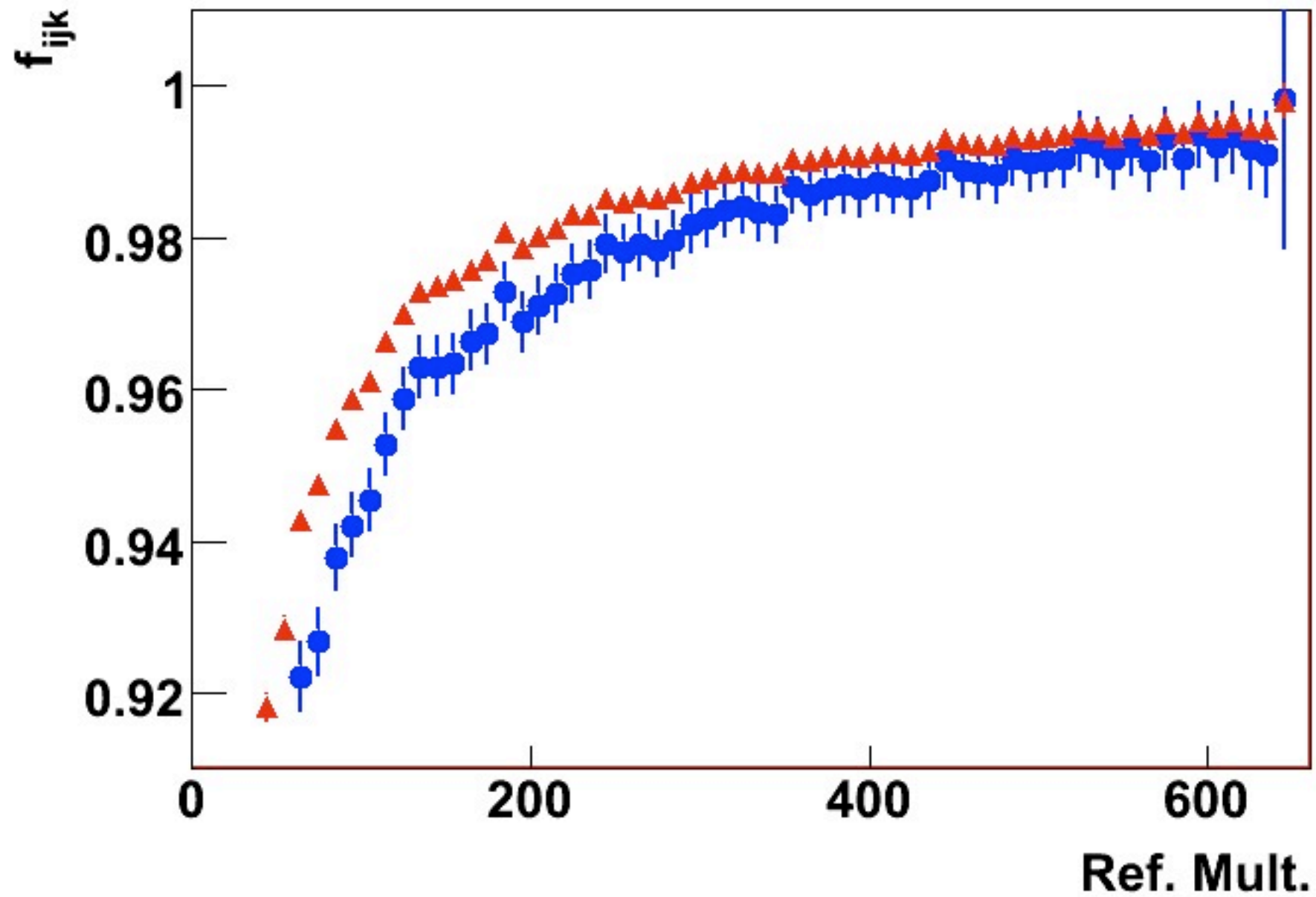
$$\frac{v_2 v_2 v_4}{(1 - f_{ij}) v_2 v_2} = 0.5$$

$$\frac{v_2 v_2 v_4}{(1 - f_{ij}) v_2 v_2} = 2$$

$$f_{ij} < 1$$

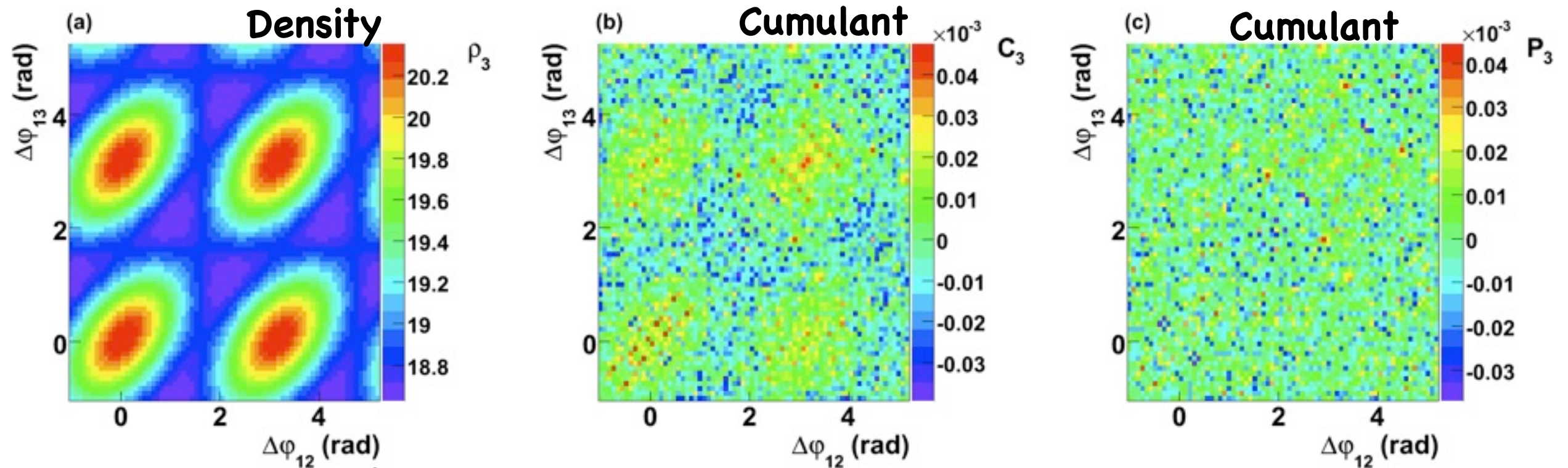
$$v_4 = 0$$

Au+Au 200 GeV 3-20 GeV, 1-2 GeV

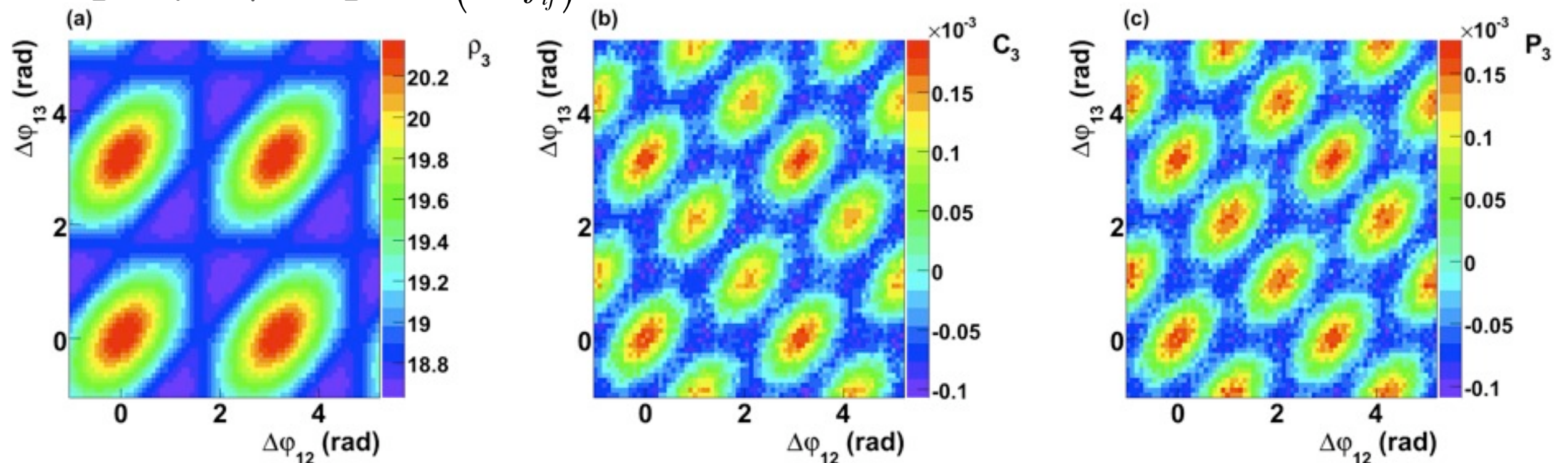


Flow – Simulations C_3 vs P_3

$$v_2 > 0; v_4 = 0$$



$$v_2 > 0; v_4 = v_2^2 \quad (1 - f_{ij}) = 0.01$$



Jet - Flow Cross Term - A toy Model

Jet Profile: $P_{Jet}(\varphi_i | \phi) = G(\varphi_i - \phi; \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(\varphi_i - \phi)^2}{2\sigma_i^2}\right)$

Particle angle φ_i
Jet axis ϕ
Jet width σ_i

Jet Flow: $P_{Jet-Axis}(\phi | \psi) = 1 + 2 \sum_n v_n(jet) \cos(n(\phi - \psi))$

Reaction Plane ψ

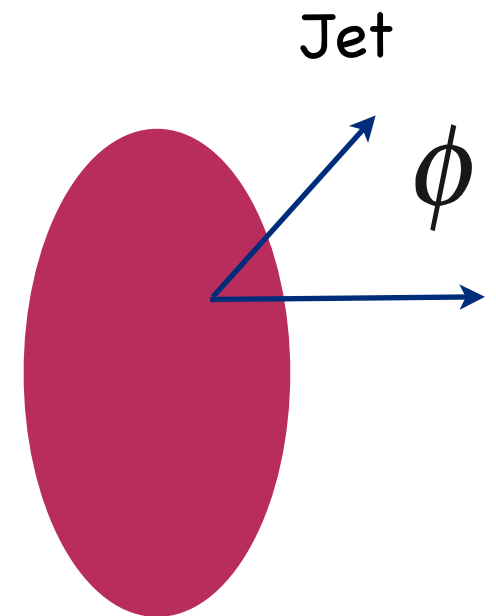
Background Flow: $P_{BCKG}(\varphi_k | \psi) = 1 + 2 \sum_n v_n(bckg) \cos(n(\varphi_k - \psi))$

3-Cumulant: $C_3(\varphi_i, \varphi_j, \varphi_k)_{FJ} = 2(2\pi)^{-1} \langle J \rangle \langle A_i A_j \rangle \langle B_k \rangle$

Avg. Jet/event $\langle J \rangle$
Avg. Associates/jet $\langle A_i A_j \rangle$
Avg. Bckg/jet $\langle B_k \rangle$

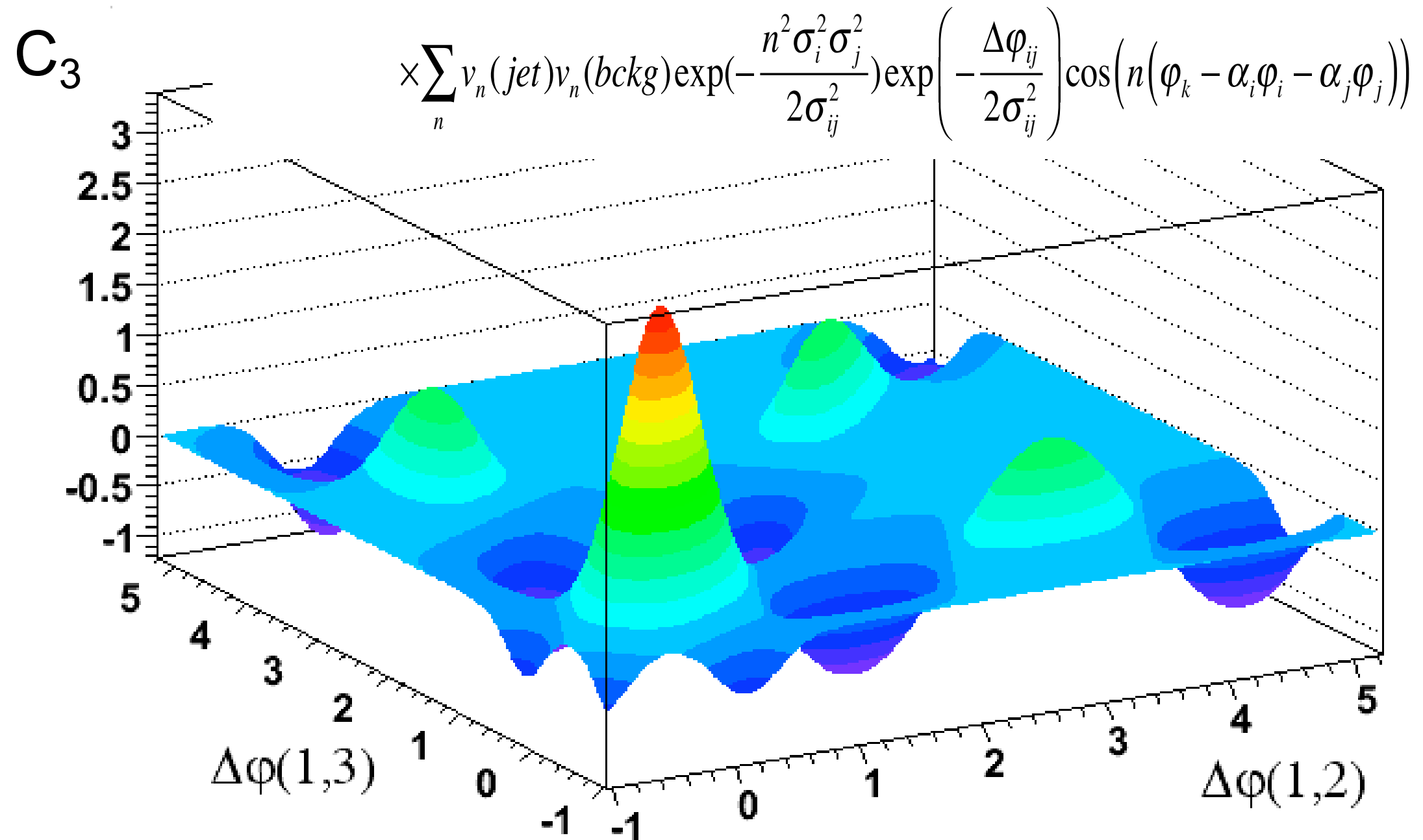
$$\times \sum_n v_n(jet) v_n(bckg) \exp\left(-\frac{n^2 \sigma_i^2 \sigma_j^2}{2\sigma_{ij}^2}\right) \exp\left(-\frac{\Delta\varphi_{ij}}{2\sigma_{ij}^2}\right) \cos\left(n\left(\varphi_k - \alpha_i \varphi_i - \alpha_j \varphi_j\right)\right)$$

$$\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2 \quad \alpha_i = \frac{\sigma_j^2}{\sigma_i^2 + \sigma_j^2}$$



Jet – Flow Cross Term – A toy Model

$$C_3(\varphi_i, \varphi_j, \varphi_k)_{FJ} = 2(2\pi)^{-1} \langle J \rangle \langle A_i A_j \rangle \langle B_k \rangle$$



Momentum/Energy Conservation

- Momentum/Energy Conservation involves all particles.
- Implies: All particles are correlated.
- Correlations exist at all orders...

Momentum Conservation Induced Correlations

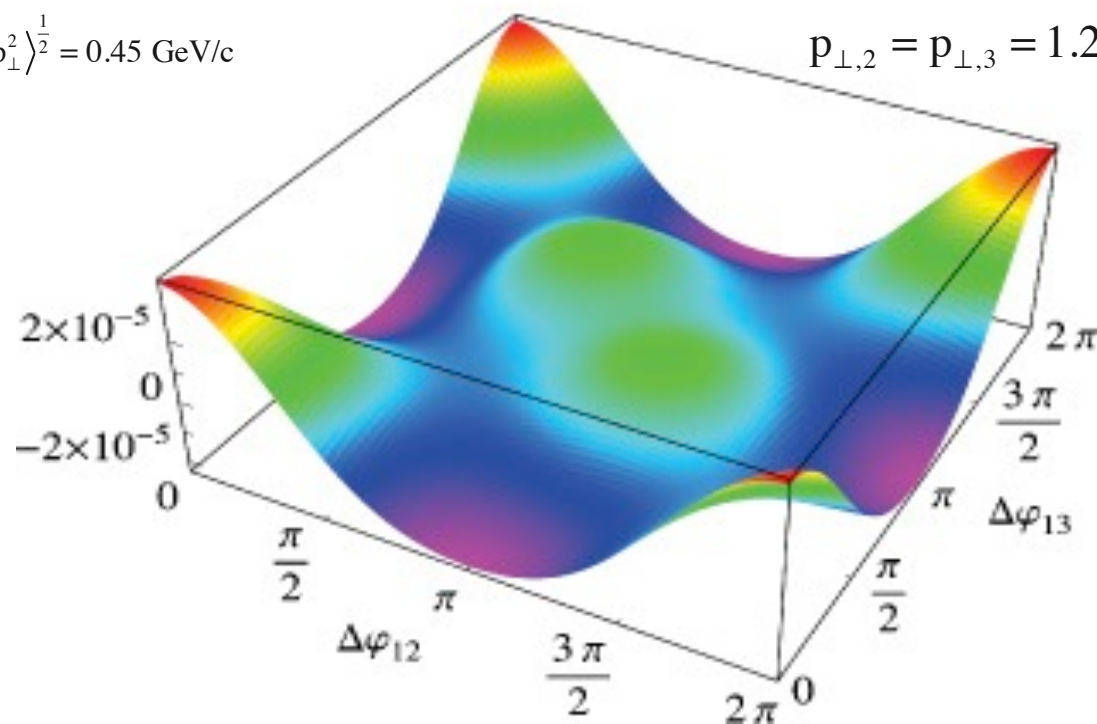
- N. Borghini, PRC 75, 21904(R) (2007).
- 3-cumulant scales as $1/N^2$.
- Assumes single-particle distribution is isotropic
- Also see Zbigniew Chajecki and Mike Lisa, arXiv:0807.3569

$$f_c(\mathbf{p}_{T1}, \mathbf{p}_{T2}) = -\frac{2 \mathbf{p}_{T1} \cdot \mathbf{p}_{T2}}{N \langle \mathbf{p}_T^2 \rangle}, \quad (1)$$

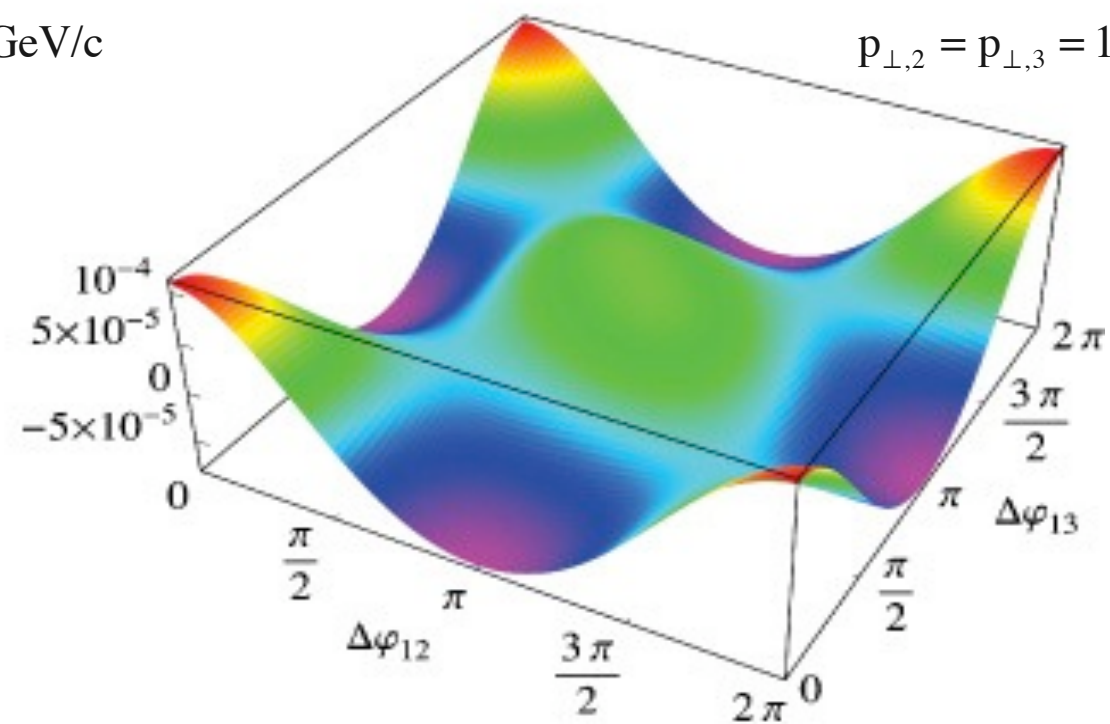
$$f_c(\mathbf{p}_{T1}, \mathbf{p}_{T2}, \mathbf{p}_{T3}) = -\frac{2}{N^2 \langle \mathbf{p}_T^2 \rangle} (\mathbf{p}_{T1} \cdot \mathbf{p}_{T2} + \mathbf{p}_{T1} \cdot \mathbf{p}_{T3} + \mathbf{p}_{T2} \cdot \mathbf{p}_{T3}) + \frac{4}{N^2 \langle \mathbf{p}_T^2 \rangle^2} [(\mathbf{p}_{T1} \cdot \mathbf{p}_{T2}) \times (\mathbf{p}_{T1} \cdot \mathbf{p}_{T3}) + (\mathbf{p}_{T1} \cdot \mathbf{p}_{T2})(\mathbf{p}_{T2} \cdot \mathbf{p}_{T3}) + (\mathbf{p}_{T1} \cdot \mathbf{p}_{T3})(\mathbf{p}_{T2} \cdot \mathbf{p}_{T3})], \quad (2)$$

$N = 8000$
 $\langle p_\perp^2 \rangle^{\frac{1}{2}} = 0.45 \text{ GeV/c}$

$p_{\perp,1} = 3.2 \text{ GeV/c}$
 $p_{\perp,2} = p_{\perp,3} = 1.2 \text{ GeV/c}$

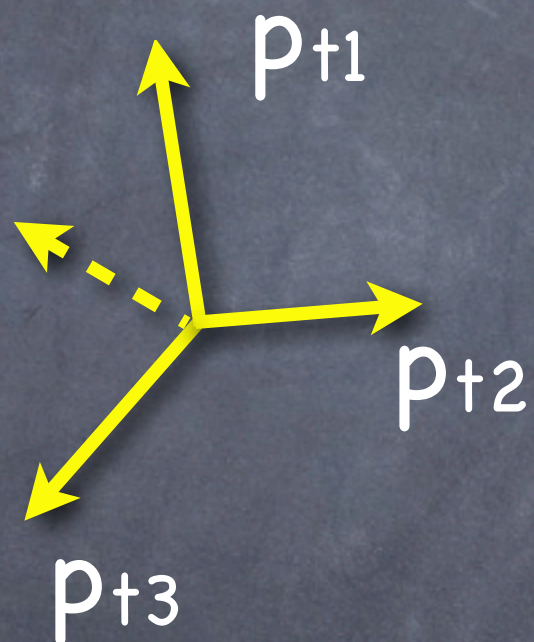


$p_{\perp,1} = 6 \text{ GeV/c}$
 $p_{\perp,2} = p_{\perp,3} = 1.2 \text{ GeV/c}$

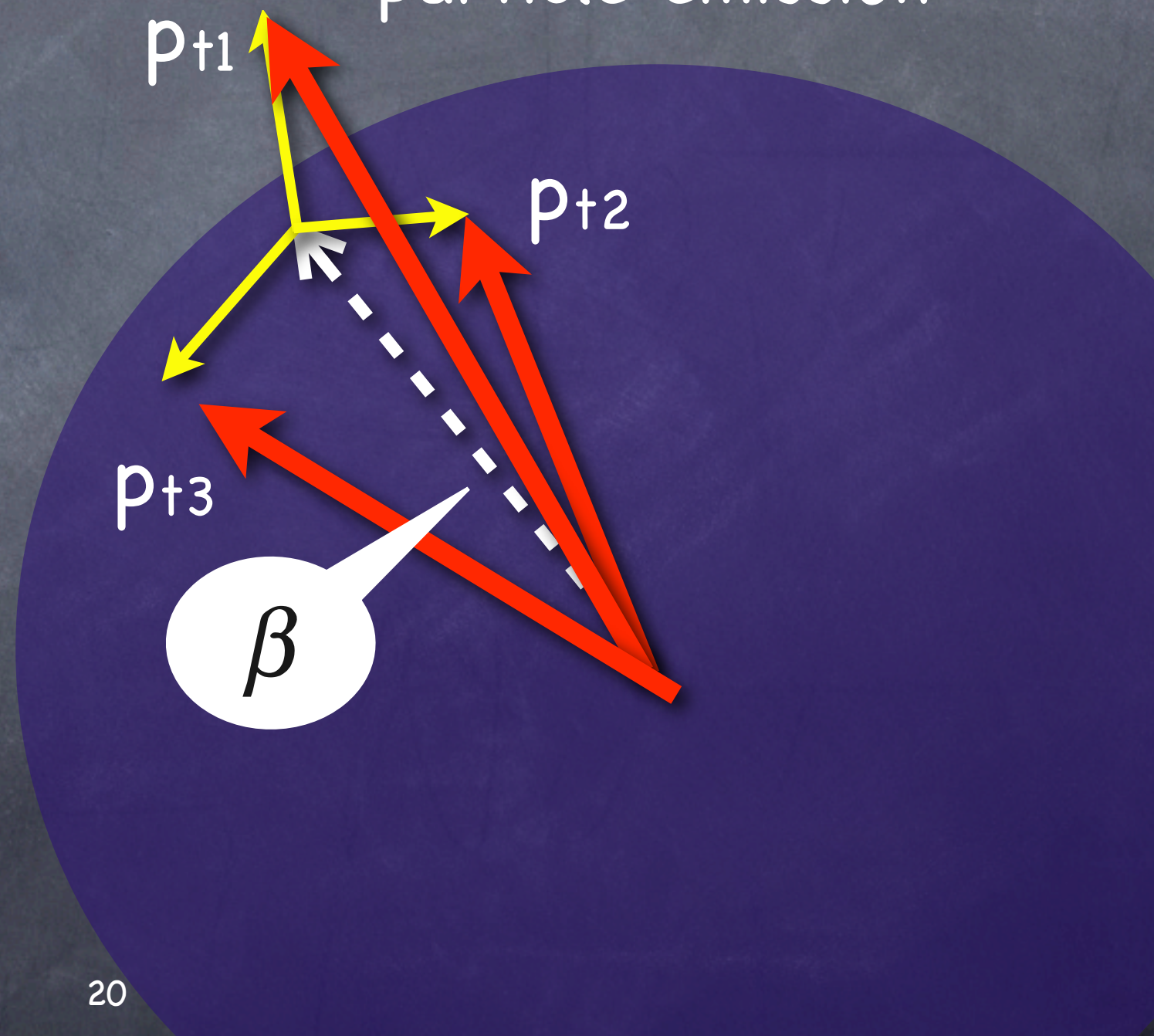


Radial Flow Induced Correlations

In vacuum particle
emission



Radially boosted
particle emission

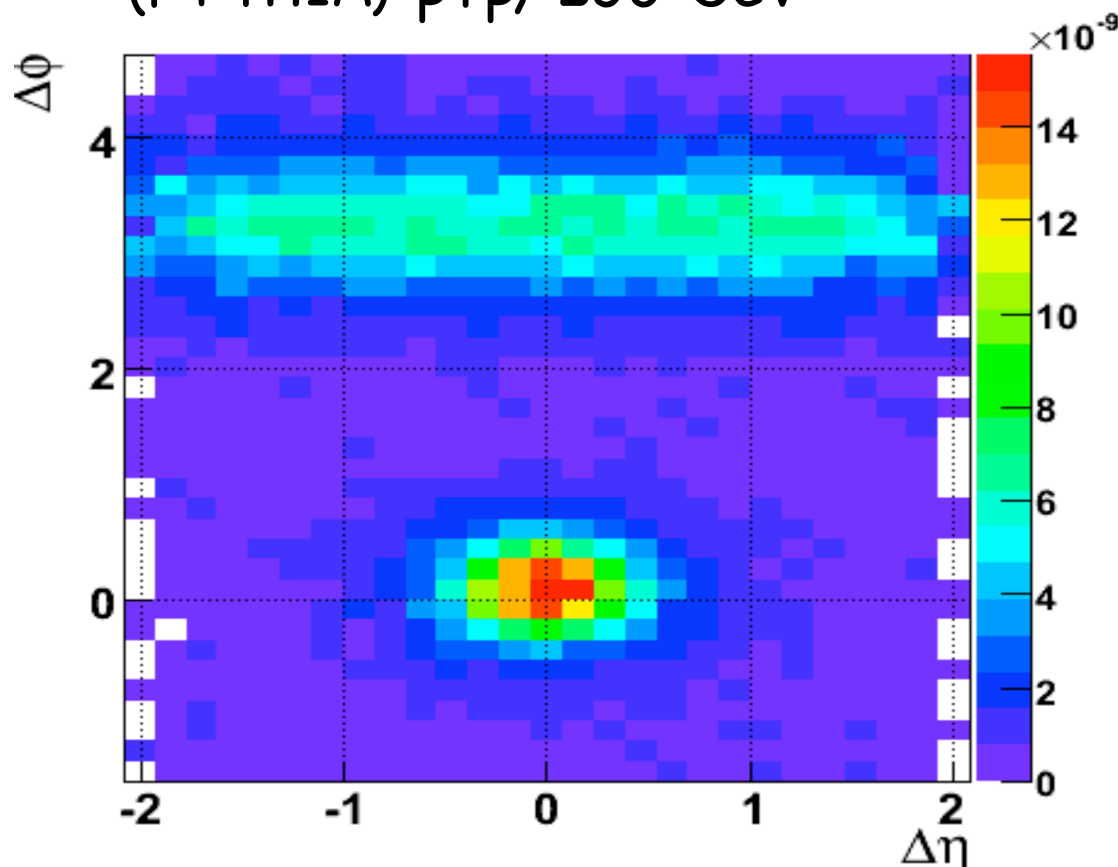


Study of Radial Flow Effects with PYTHIA

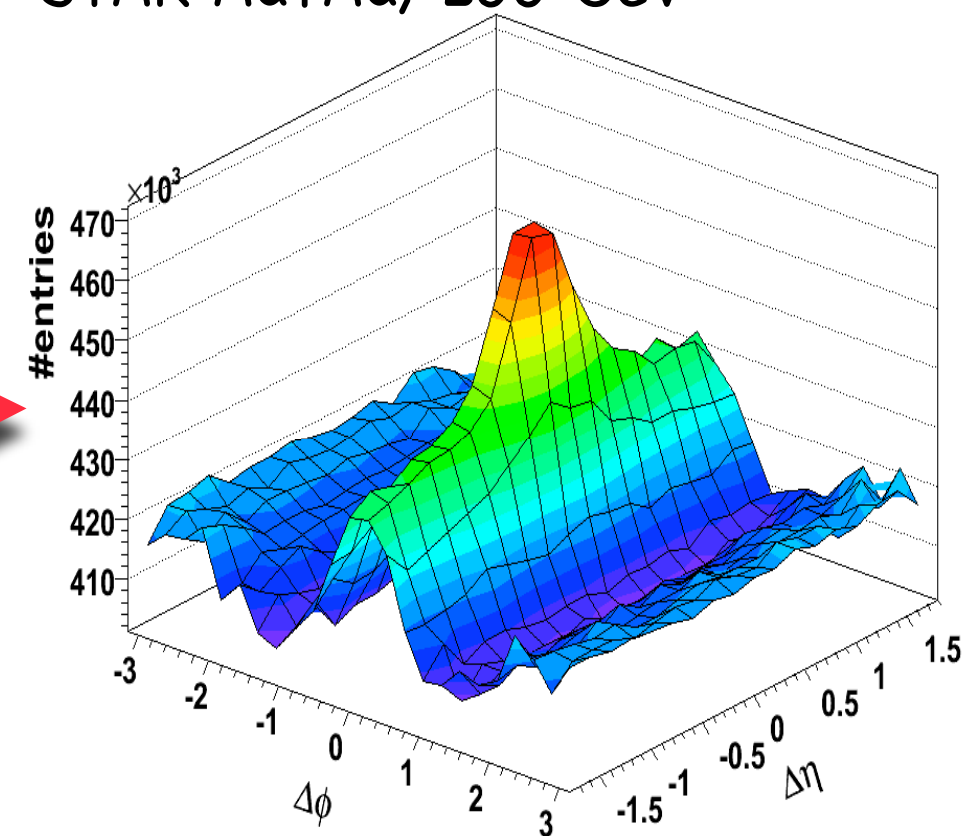
C.P., S. Gavin, S. Voloshin, Nucl. Phys. A802, 107 (2008).

- Motivation: Can the ridge found in Au+Au $\Delta\eta$ vs $\Delta\phi$ correlations be due to radial flow?
- Use PYTHIA p+p events (200 GeV) with radial boost to simulate the effect of radial flow.
- Particles emitted by one p+p interaction are all boosted in the same radial direction.

(PYTHIA) p+p, 200 GeV



STAR Au+Au, 200 GeV

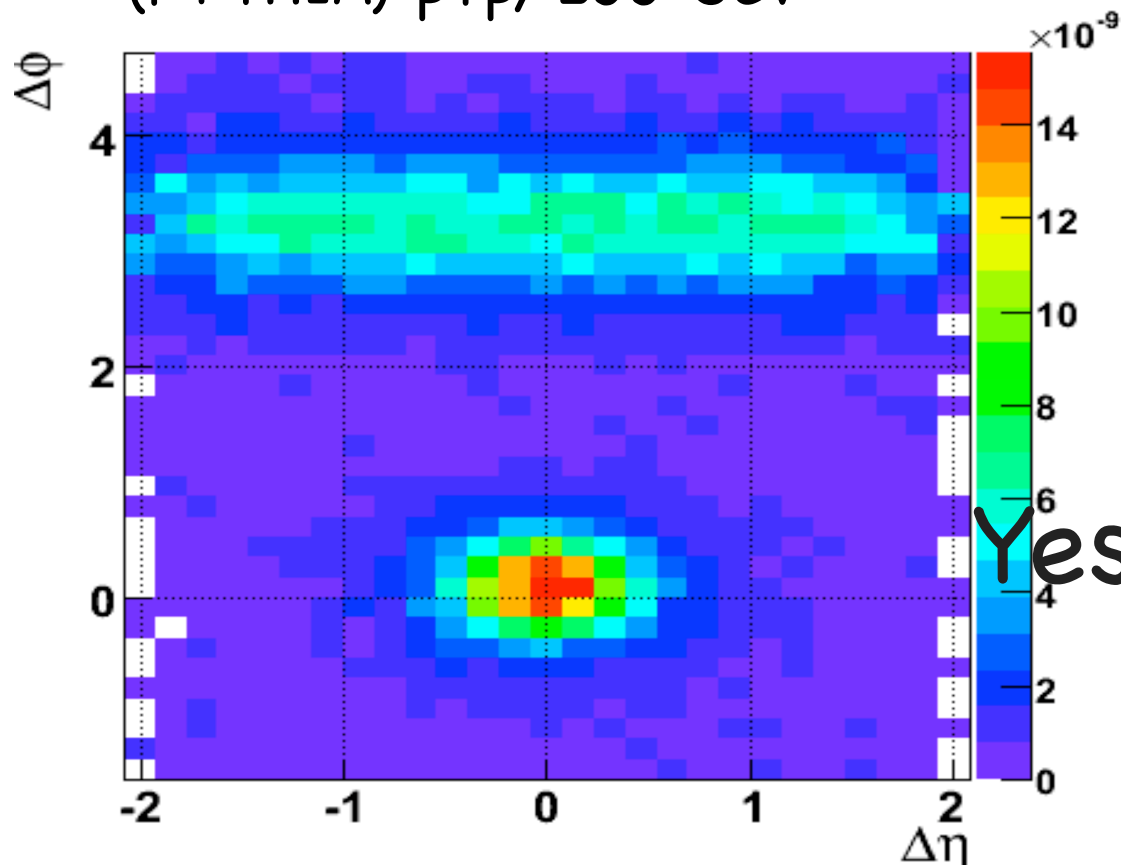


Study of Radial Flow Effects with PYTHIA

C.P., S. Gavin, S. Voloshin, Nucl. Phys. A802, 107 (2008).

- Motivation: Can the ridge found in Au+Au $\Delta\eta$ vs $\Delta\phi$ correlations be due to radial flow?
- Use PYTHIA p+p events (200 GeV) with radial boost to simulate the effect of radial flow.
- Particles emitted by one p+p interaction are all boosted in the same radial direction.

(PYTHIA) p+p, 200 GeV

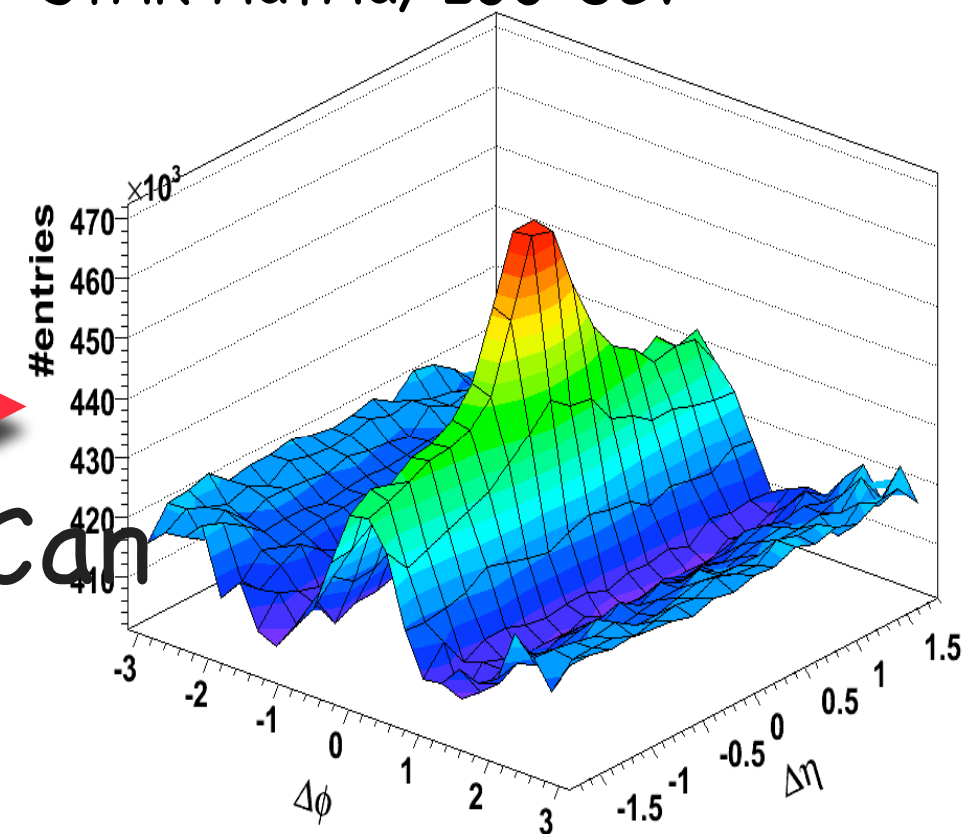


?



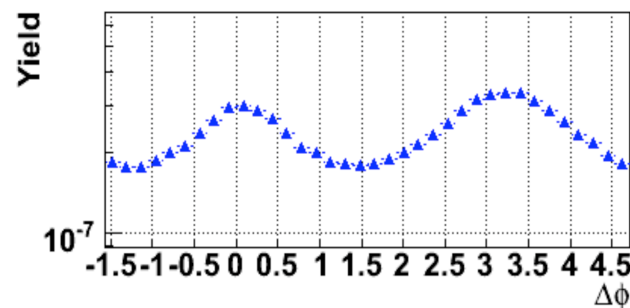
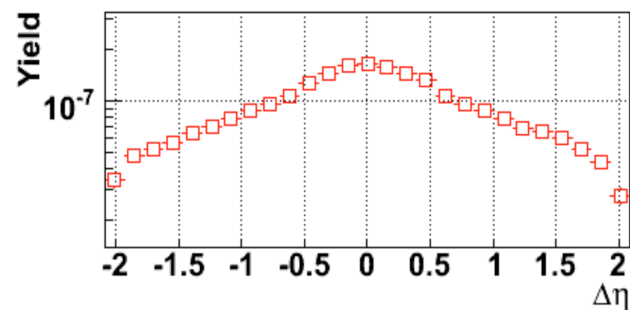
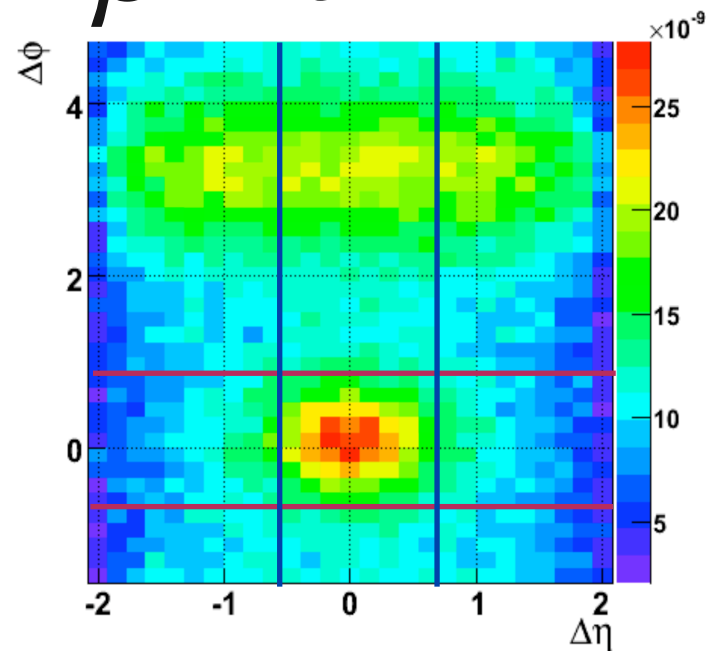
Yes We Can

STAR Au+Au, 200 GeV

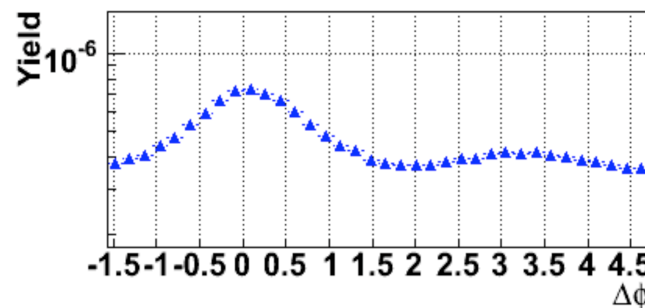
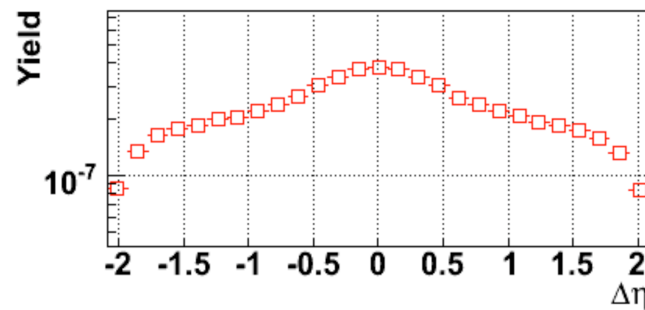
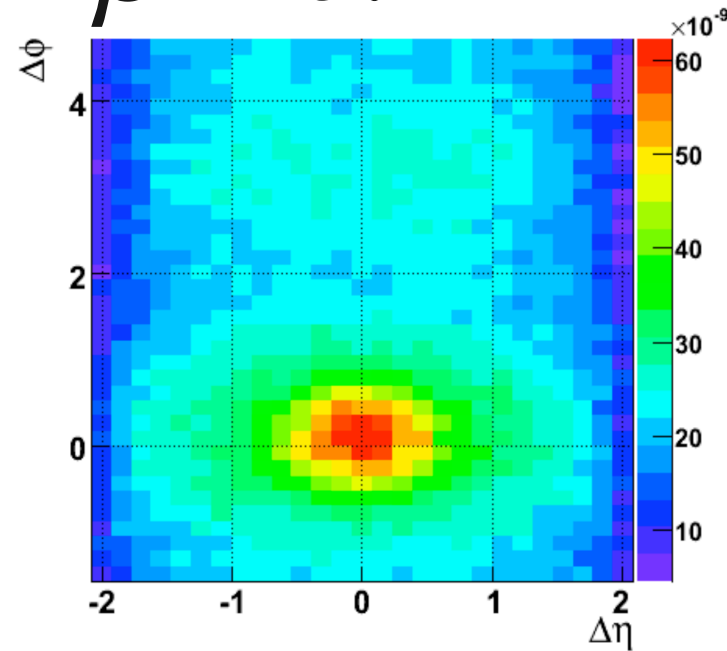


PYTHIA p+p @ sqrt(s)=200 GeV; $3 < p_t < 20$ && $0.2 < p_t < 1$

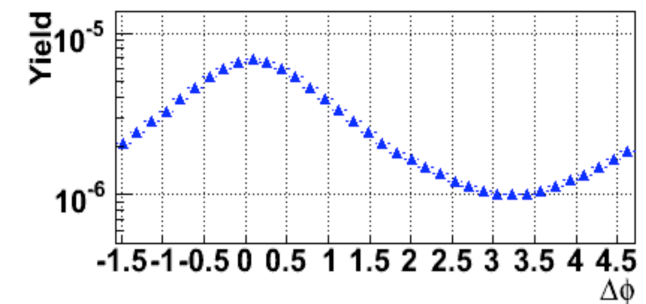
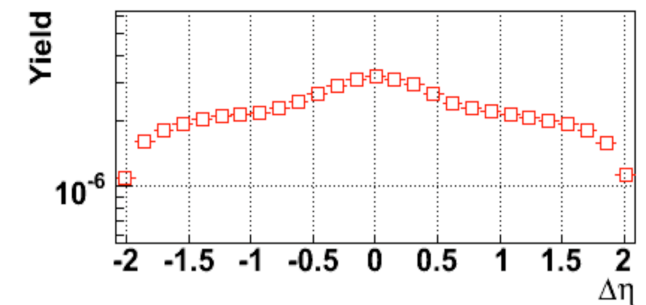
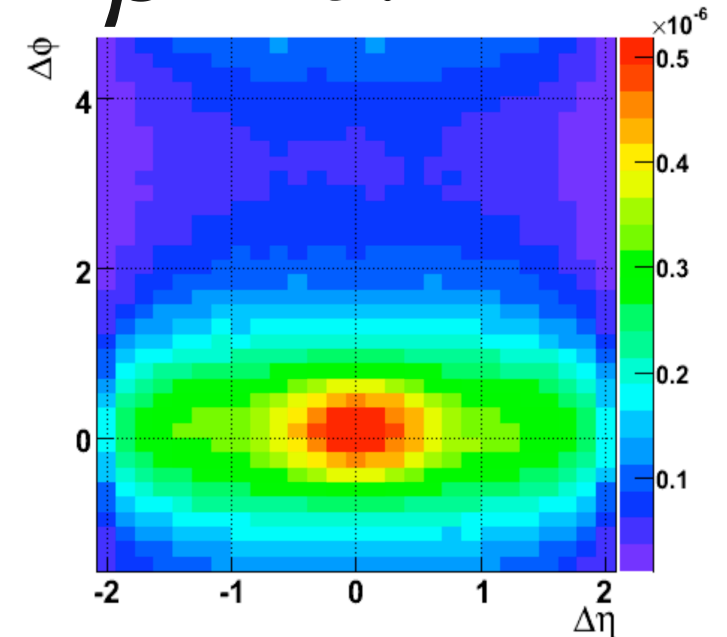
$\beta = 0$



$\beta = 0.2$

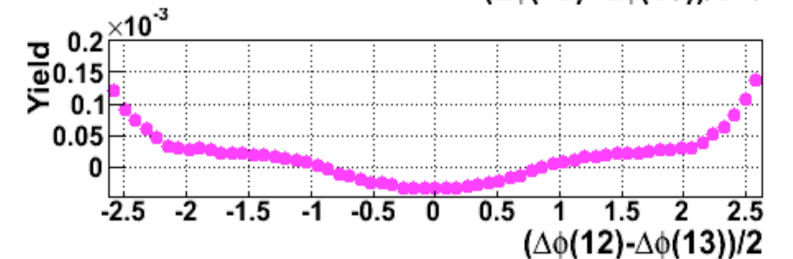
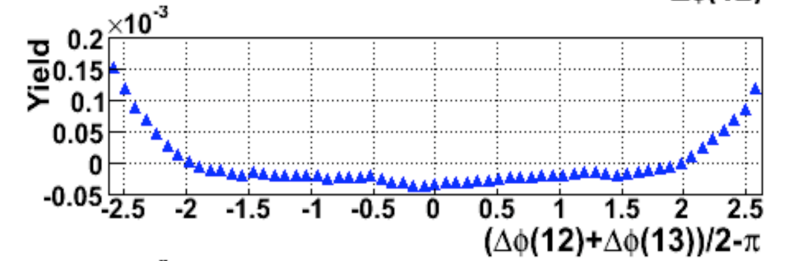
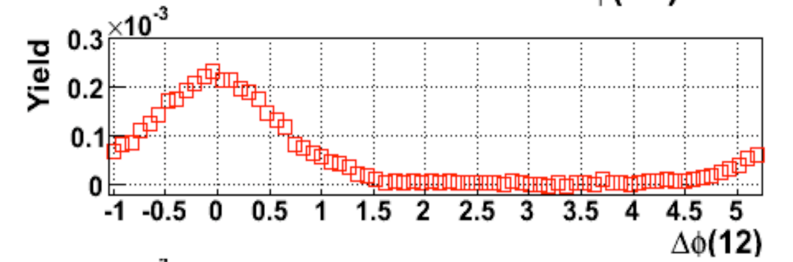
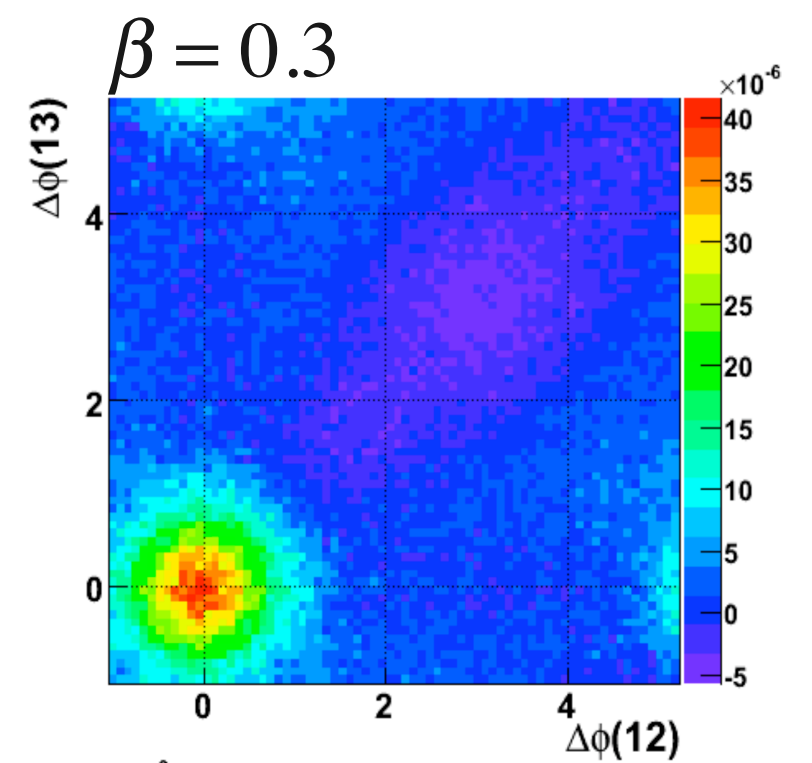
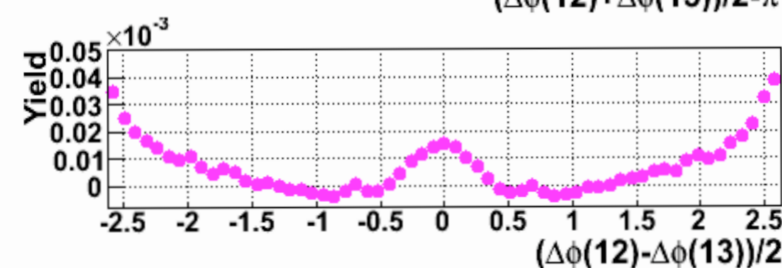
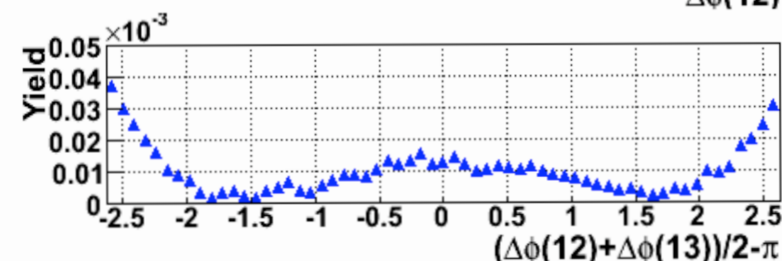
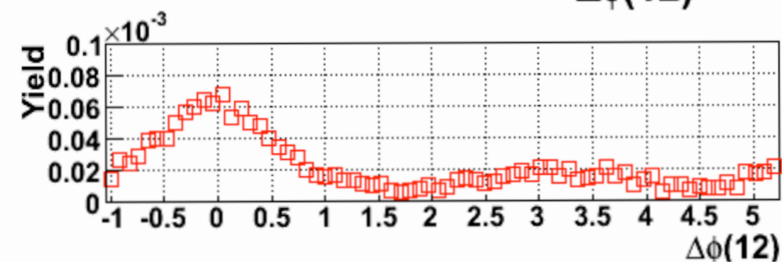
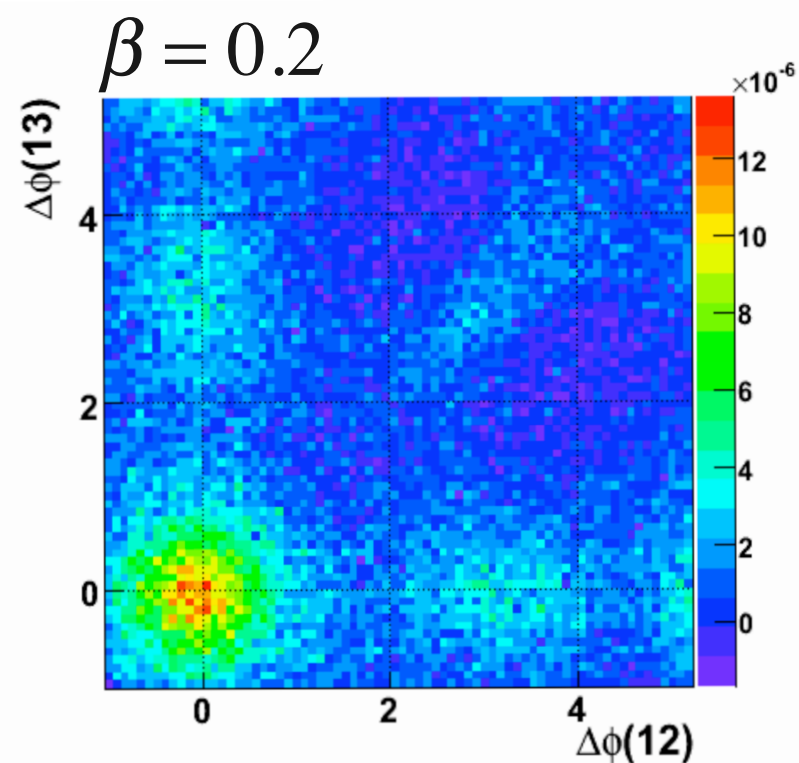
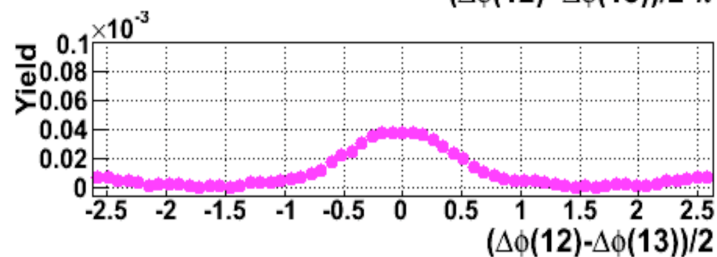
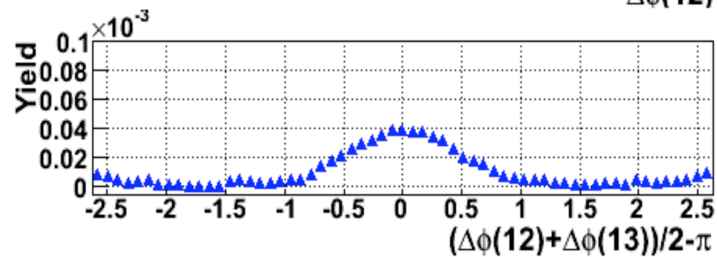
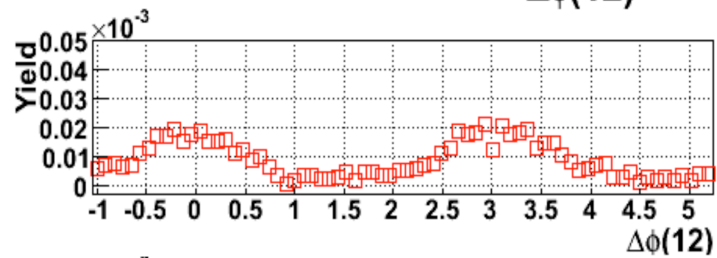
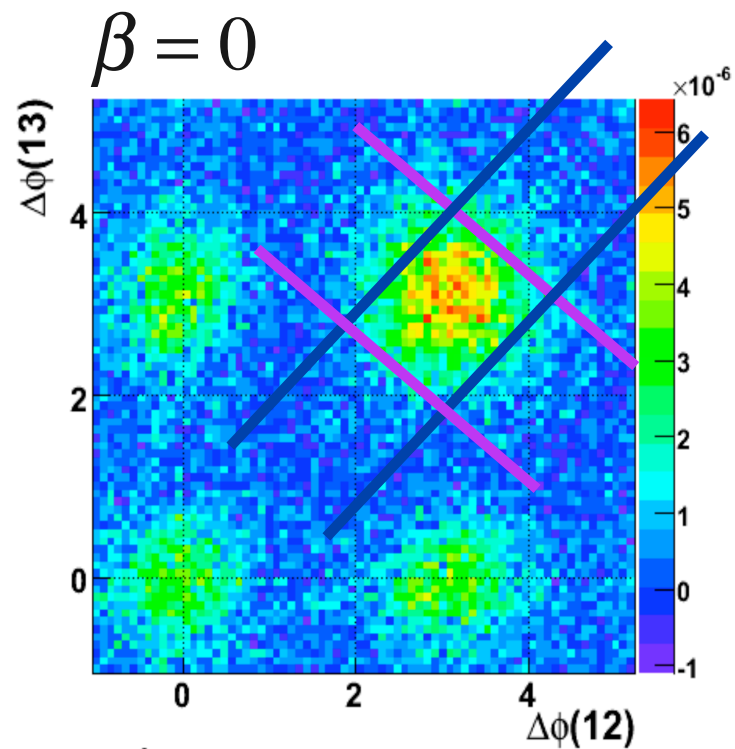


$\beta = 0.4$



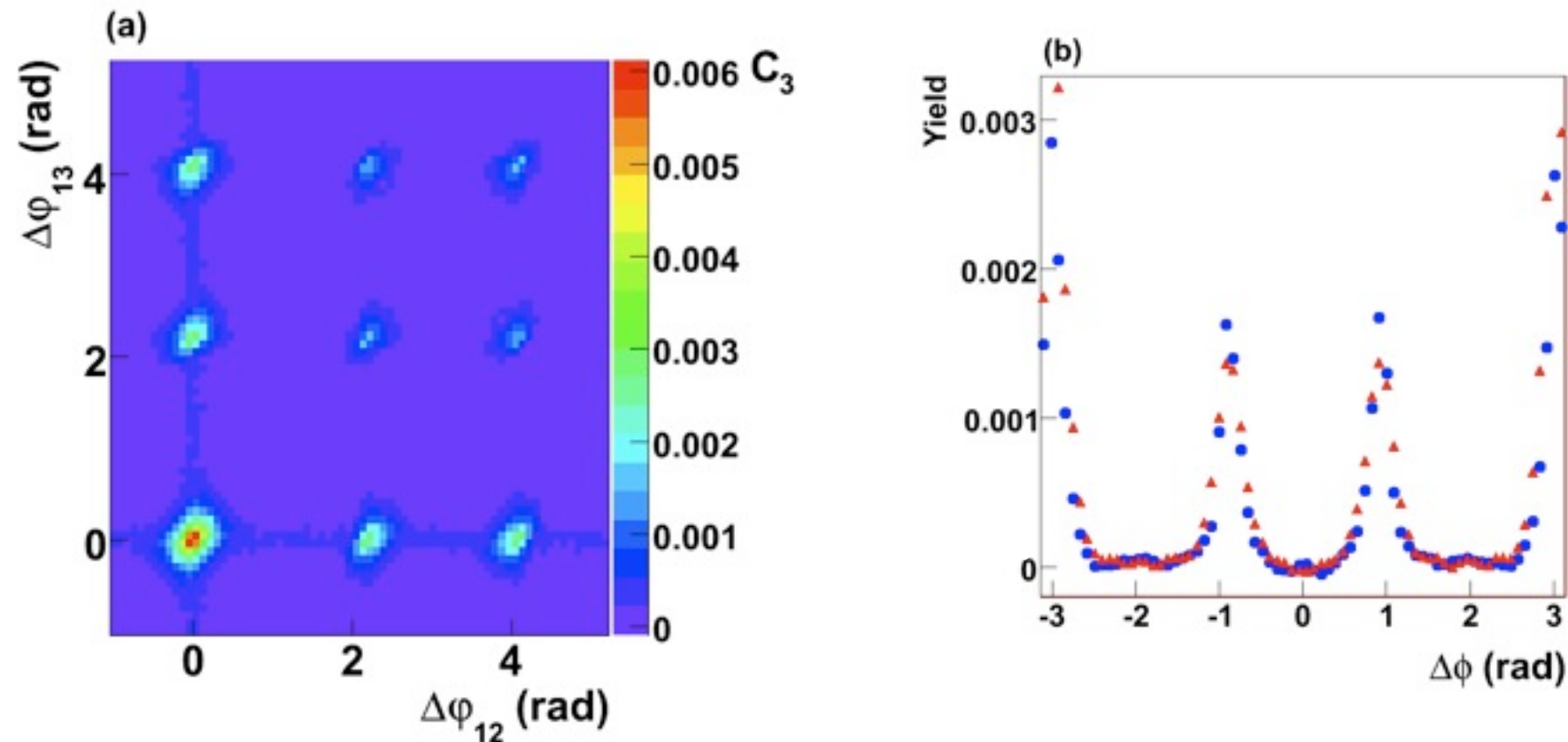
So..., what about 3-correlations??

PYTHIA p+p @ sqrt(s)=200 GeV; $3 < p_t < 20$ && $0.2 < p_t < 1$



Radial flow alters 3-cumulant

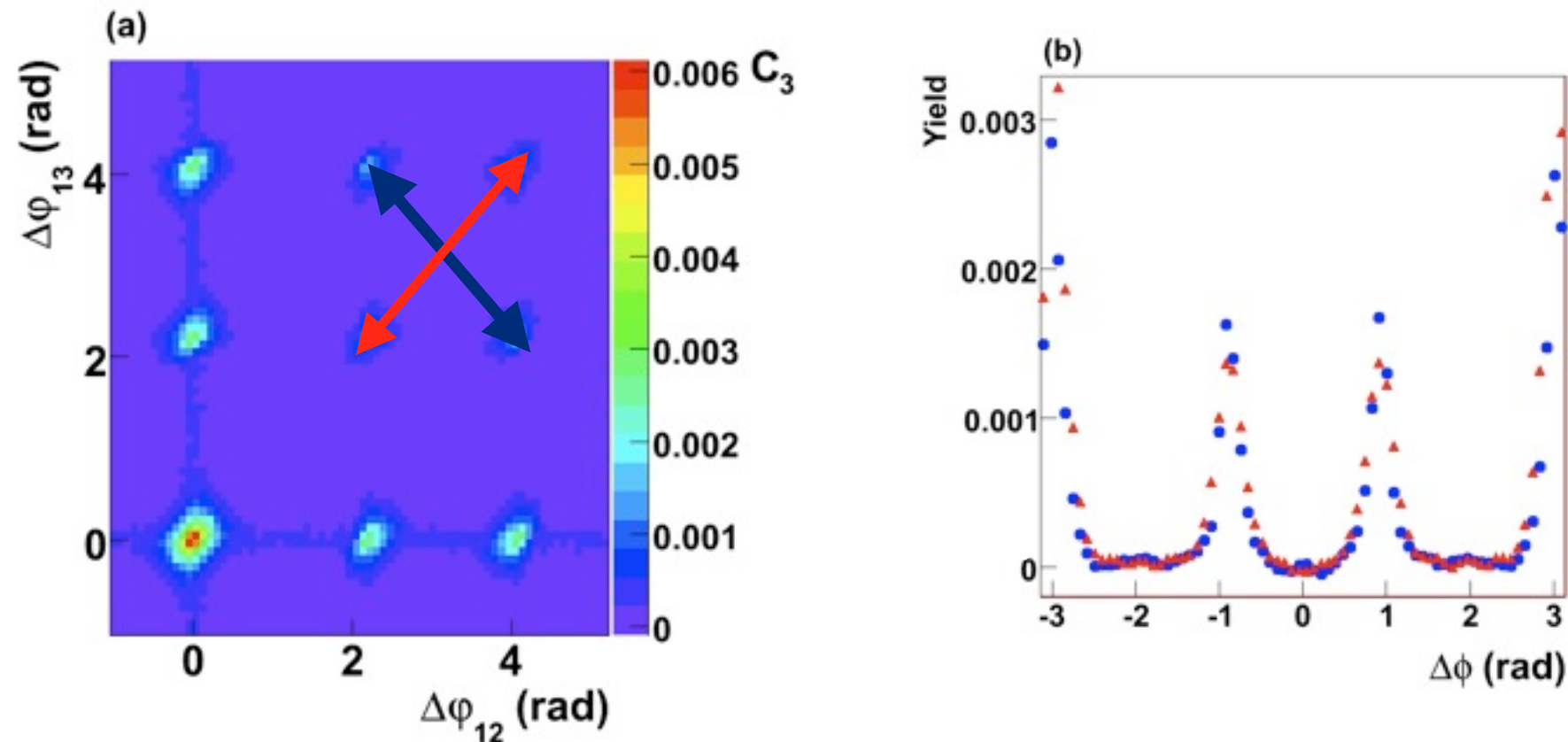
Cone Emission Simulation



Simulation: Average Multiplicities

- Particle 1 (High pt):
 - Background: 1 part/evt
 - Trigger (Jet tag): 1 part/event
- Particle 2 (Low pt):
 - Background 100 part/evt
 - Cone associates: 2 part event

Cone Emission Simulation

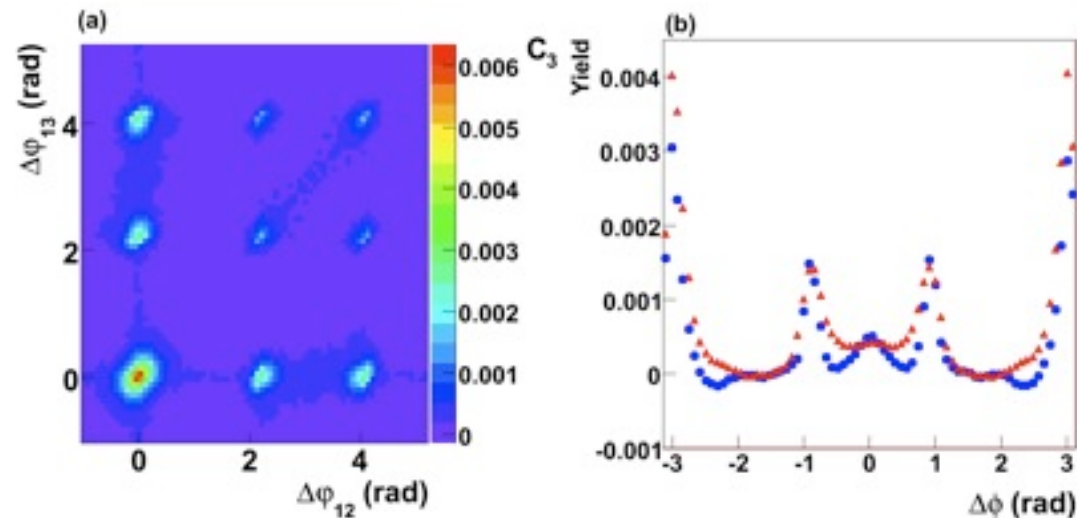


Simulation: Average Multiplicities

- Particle 1 (High pt):
 - Background: 1 part/evt
 - Trigger (Jet tag): 1 part/event
- Particle 2 (Low pt):
 - Background 100 part/evt
 - Cone associates: 2 part event

Simulation: Jet + Cone + Flow

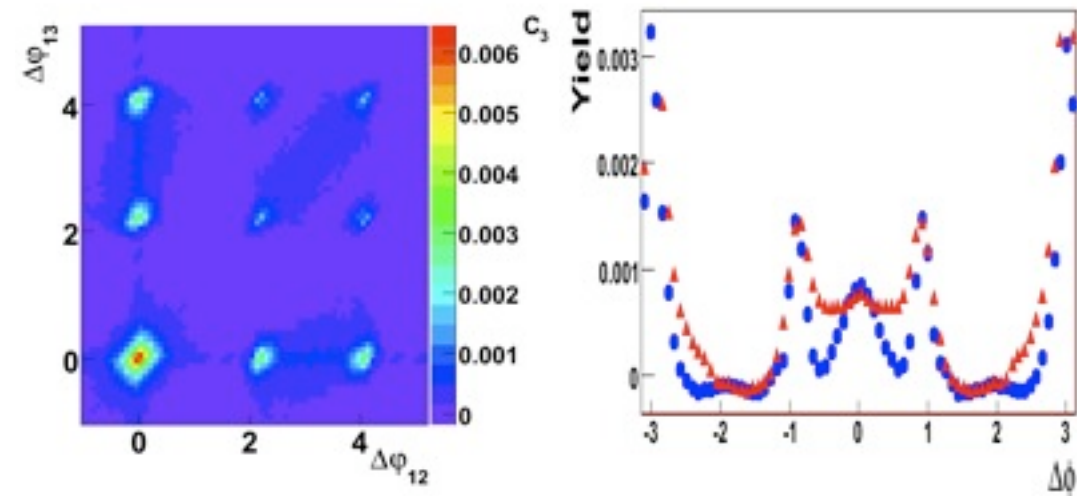
but no jet-flow



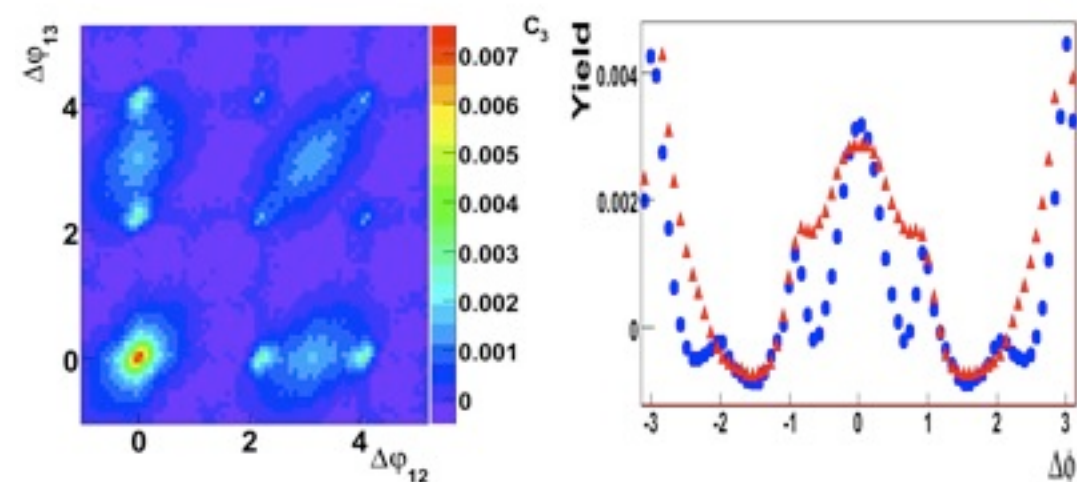
Low pt signal/bckg = 1/100

High pt signal/bckg = 1

$$v_2 = 0.1, v_4 = v_2^2$$



High pt signal/bckg = 1/2



High pt signal/bckg = 1/4

Comments (1)

- Combinatorial background is best removed using a cumulant method: no model or physics assumptions.
- v_2^2 flow is suppressed in C_3 and zero in P_3
- Its amplitude is measurable as f_{ijk} coefficients. No ZYAM needed.
- $\langle v_2 v_2 v_4 \rangle$ is irreducible. Note: $\langle v_2 v_2 v_4 \rangle \neq \langle v_2 \rangle \langle v_2 \rangle \langle v_4 \rangle$
- No simple model (currently) for momentum conservation effects
- No estimate for 3-particle decays
- Radial Flow Effects may be large

Comments (2)

- Cone Signal clearly visible for Poisson background even in the presence of (strong) flow.
- Cone Signal visible for non-Poisson background with moderate background and flow.
- Signal difficult to extract for large background & flow.
- Approximate sensitivity:

$$\frac{\langle N_1 N_2 N_3 \rangle_{\text{signal}}}{\langle N_1 N_2 N_3 \rangle_{\text{bckg}}} \geq \sim 10^{-1} v_2^4$$

STAR Data, Run 4

Cumulant Analysis

QM06, 08

Au + Au @ 200 GeV vs Centrality

Jet tag (Trigger): $3 < p_{\text{T}} < 20$ GeV

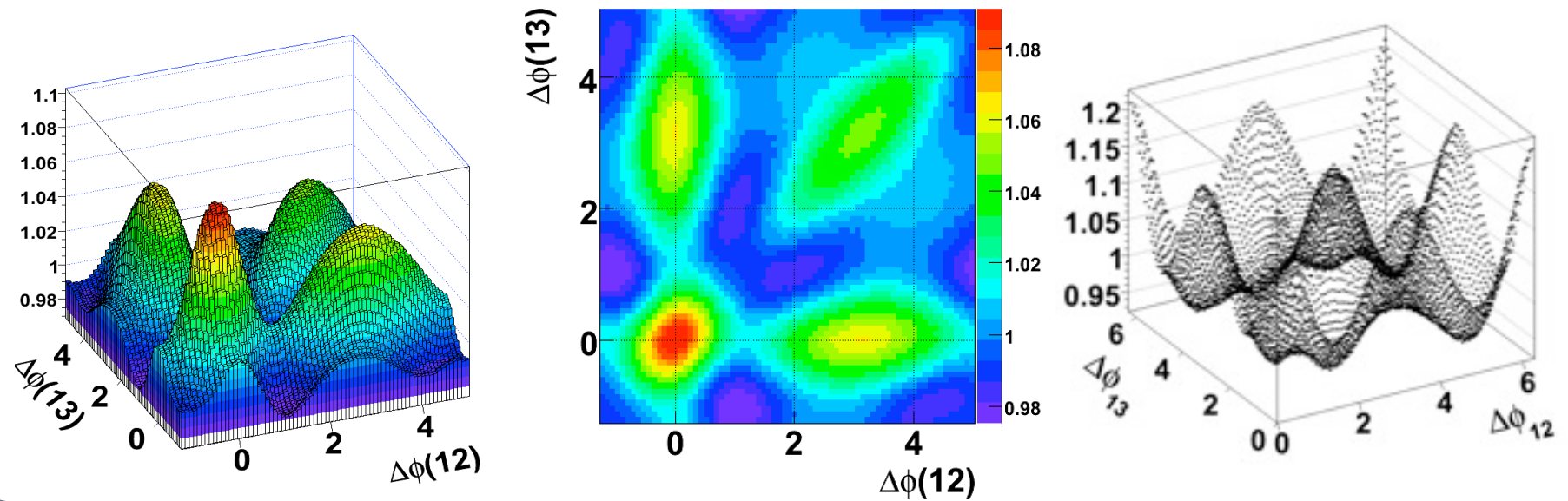
Associates (3 ranges examined)

$1 < p_{\text{T}} < 2$ GeV

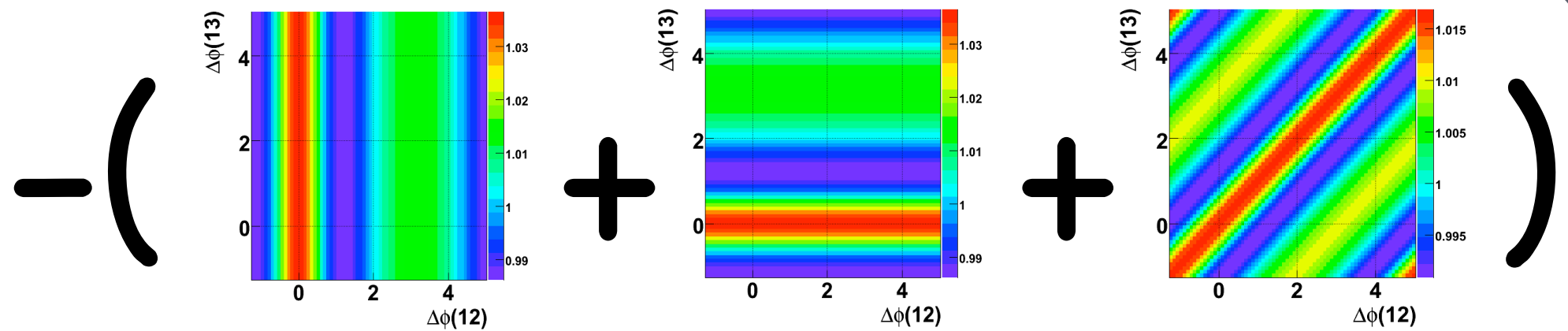
$2 < p_{\text{T}} < 3$ GeV

All particles $|\eta_{\text{tag}}| < 1$

3-Particle Density (3 views)

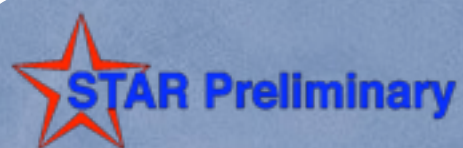
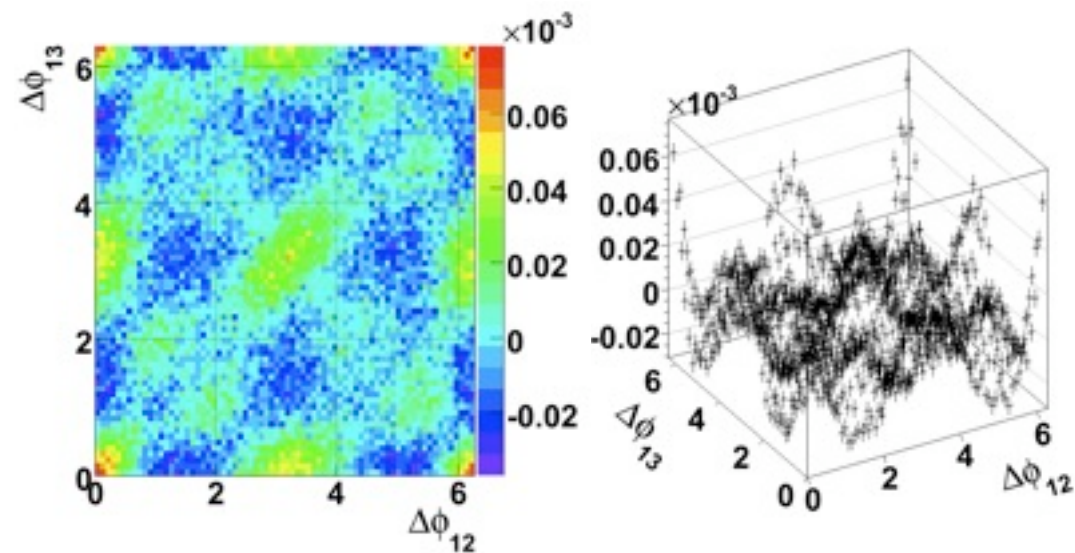


Combinatorial Terms



3-Cumulant (2 views)

=



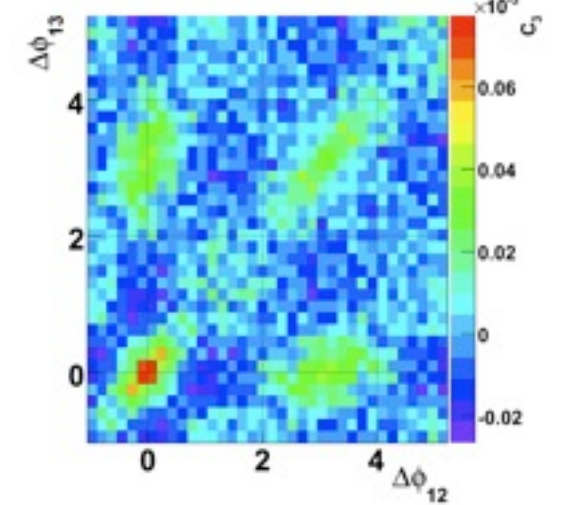
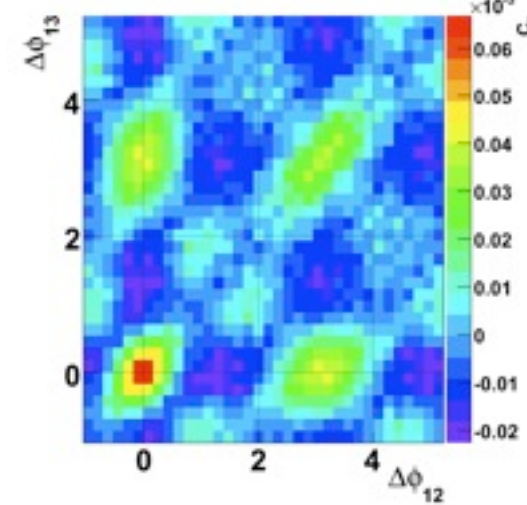
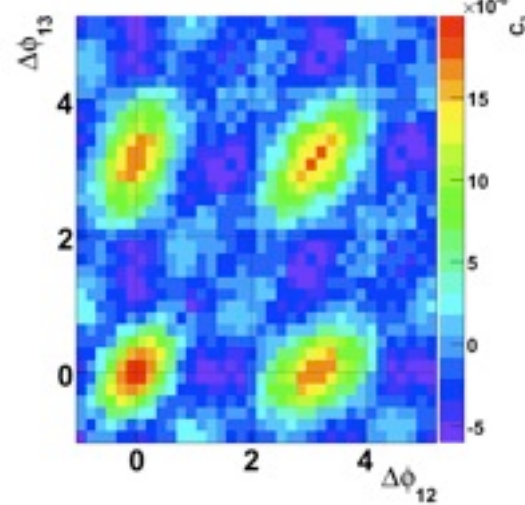
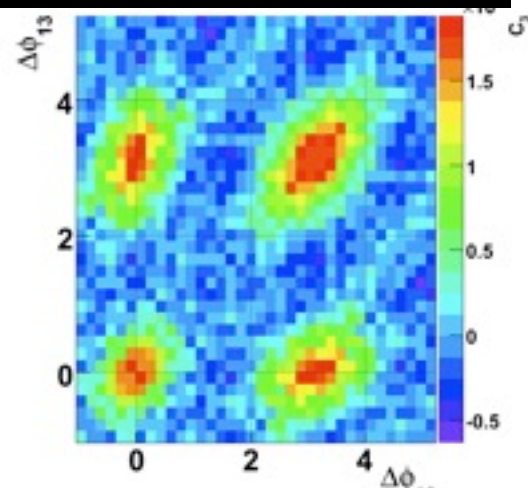
80-50%

50-30%

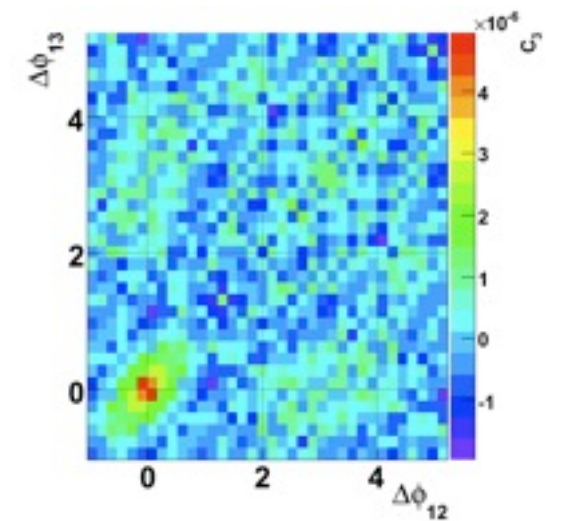
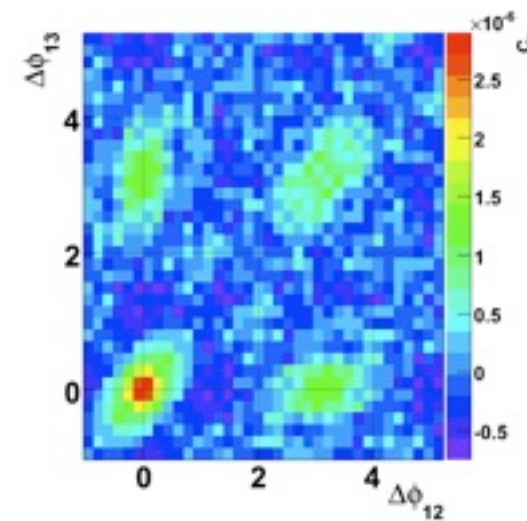
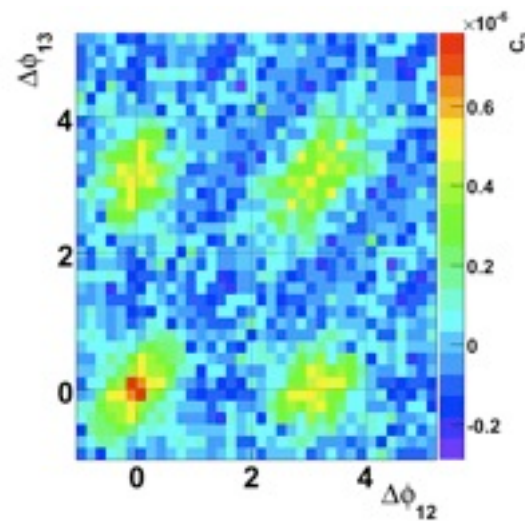
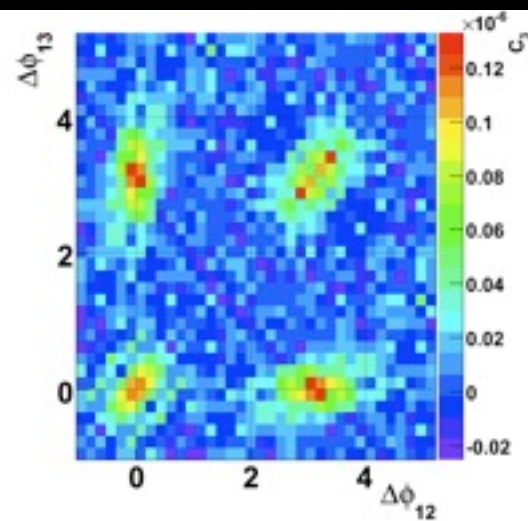
30-10%

10-0%

1.0 < pt < 2.0 GeV/c



2.0 < pt < 3.0 GeV/c



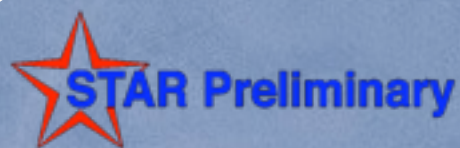
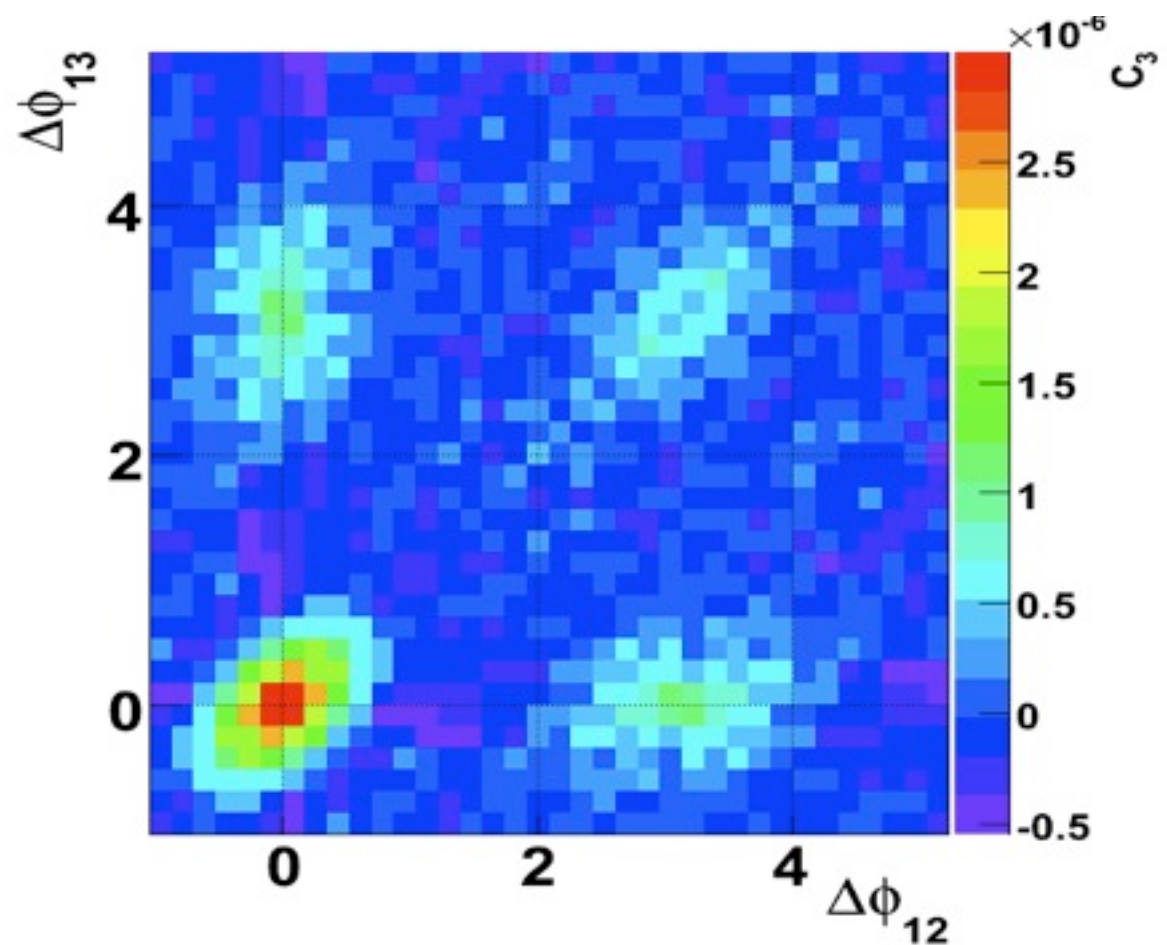
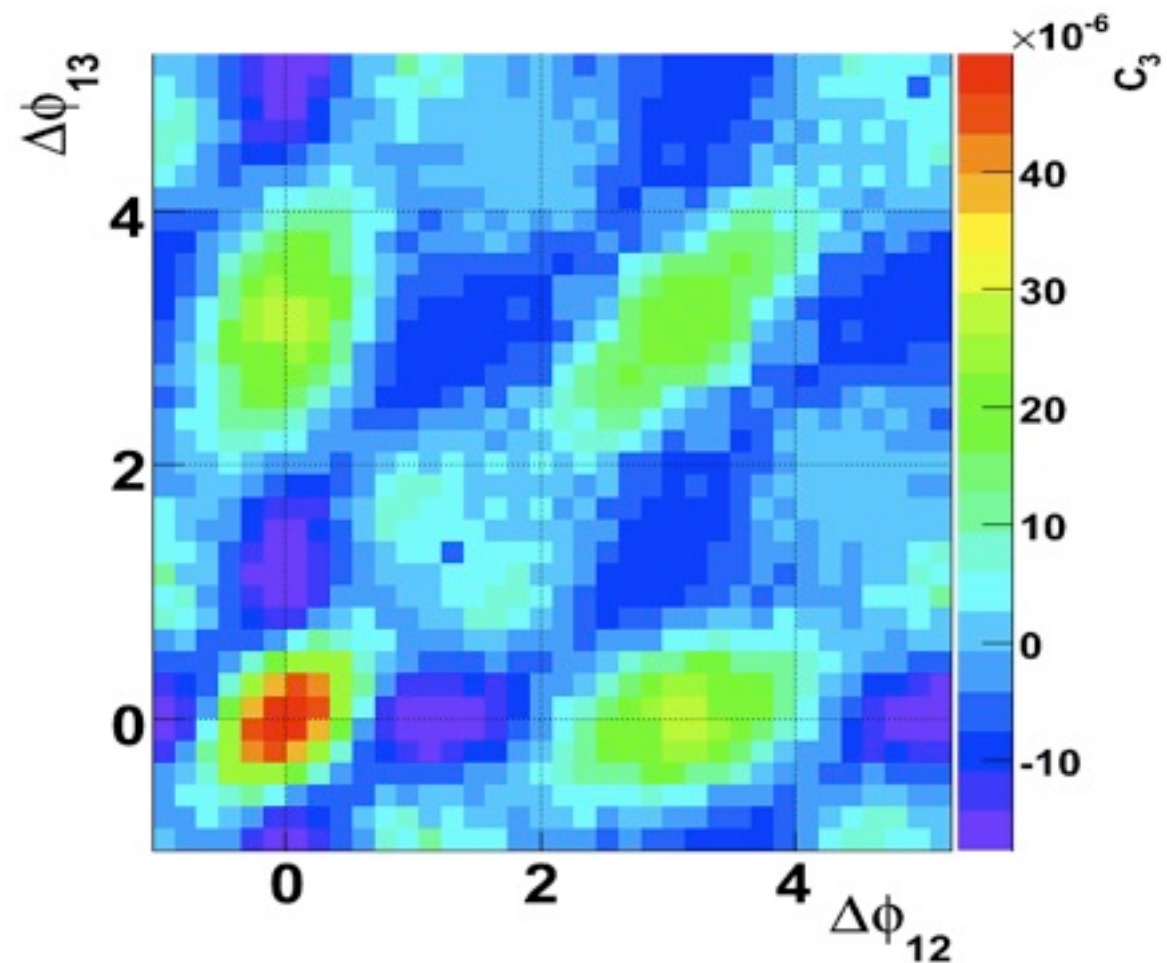
Central Trigger

$$p_{t1} > 3 \text{ GeV}/c$$

$$1 < p_{t2,3} < 2 \text{ GeV}/c$$

$$p_{t1} > 3 \text{ GeV}/c$$

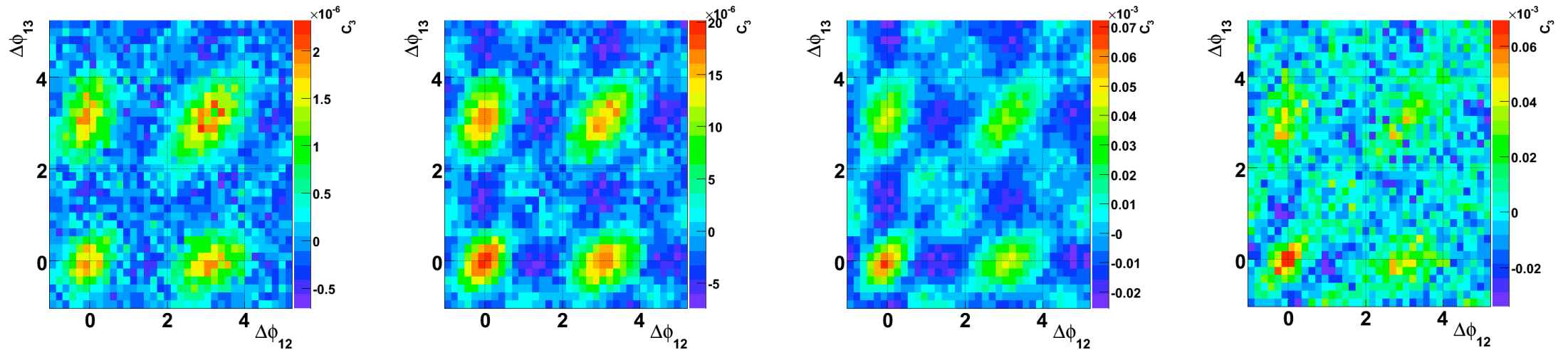
$$2 < p_{t2,3} < 3 \text{ GeV}/c$$



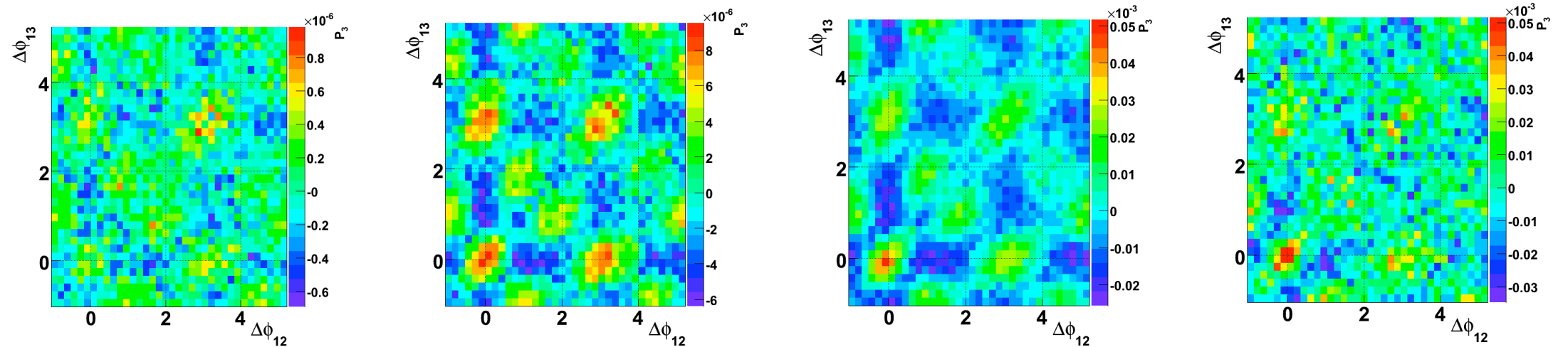
Cumulant vs Probability Cumulant

Trigger: 3 - 20 GeV, Associate: 1 - 2 GeV

C_3



P_3



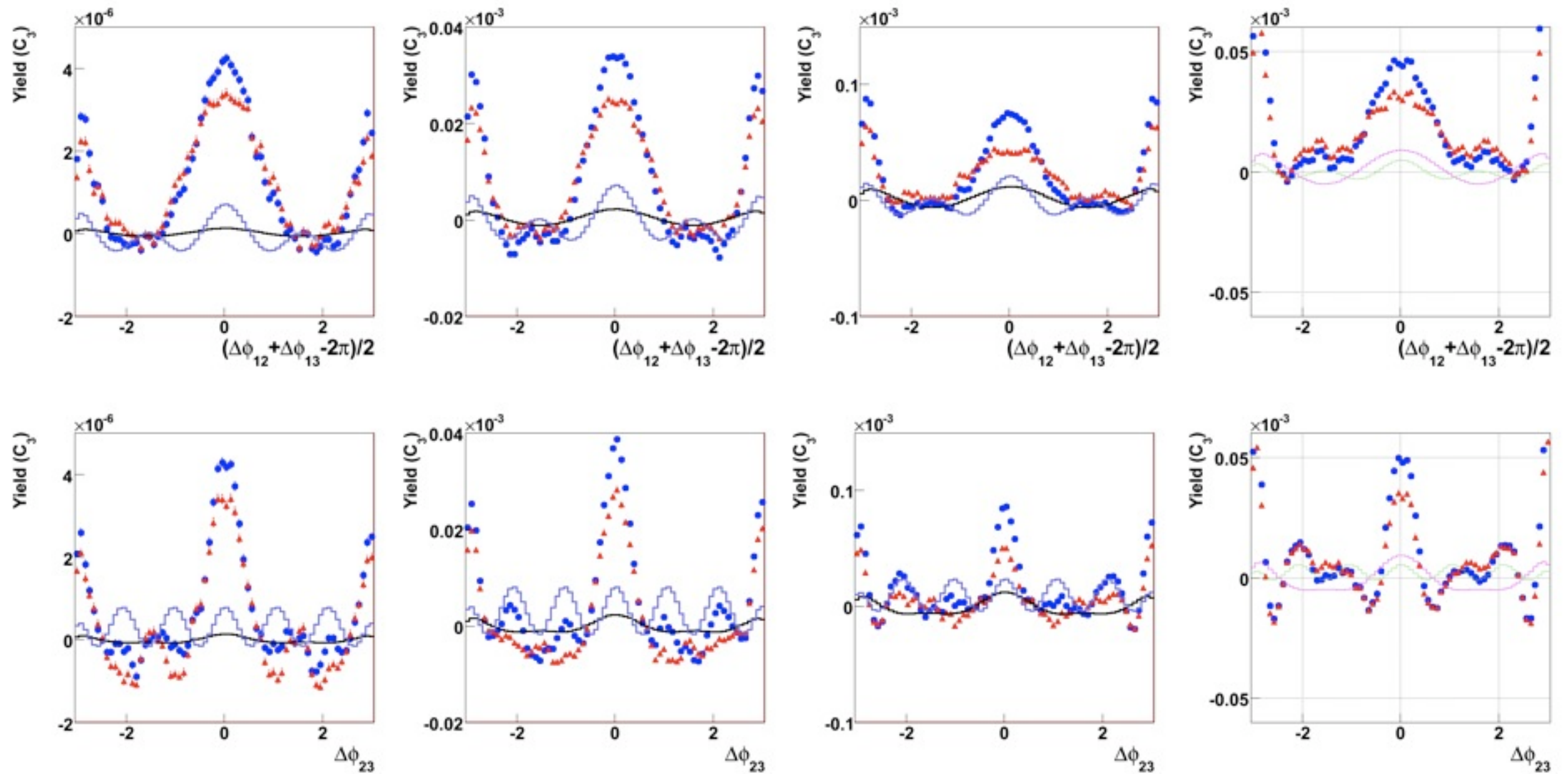
80-50%

50-30%

30-10%

10-0%

Au+Au 200 GeV 1: 3–20 GeV, 2&3: 1–2 GeV



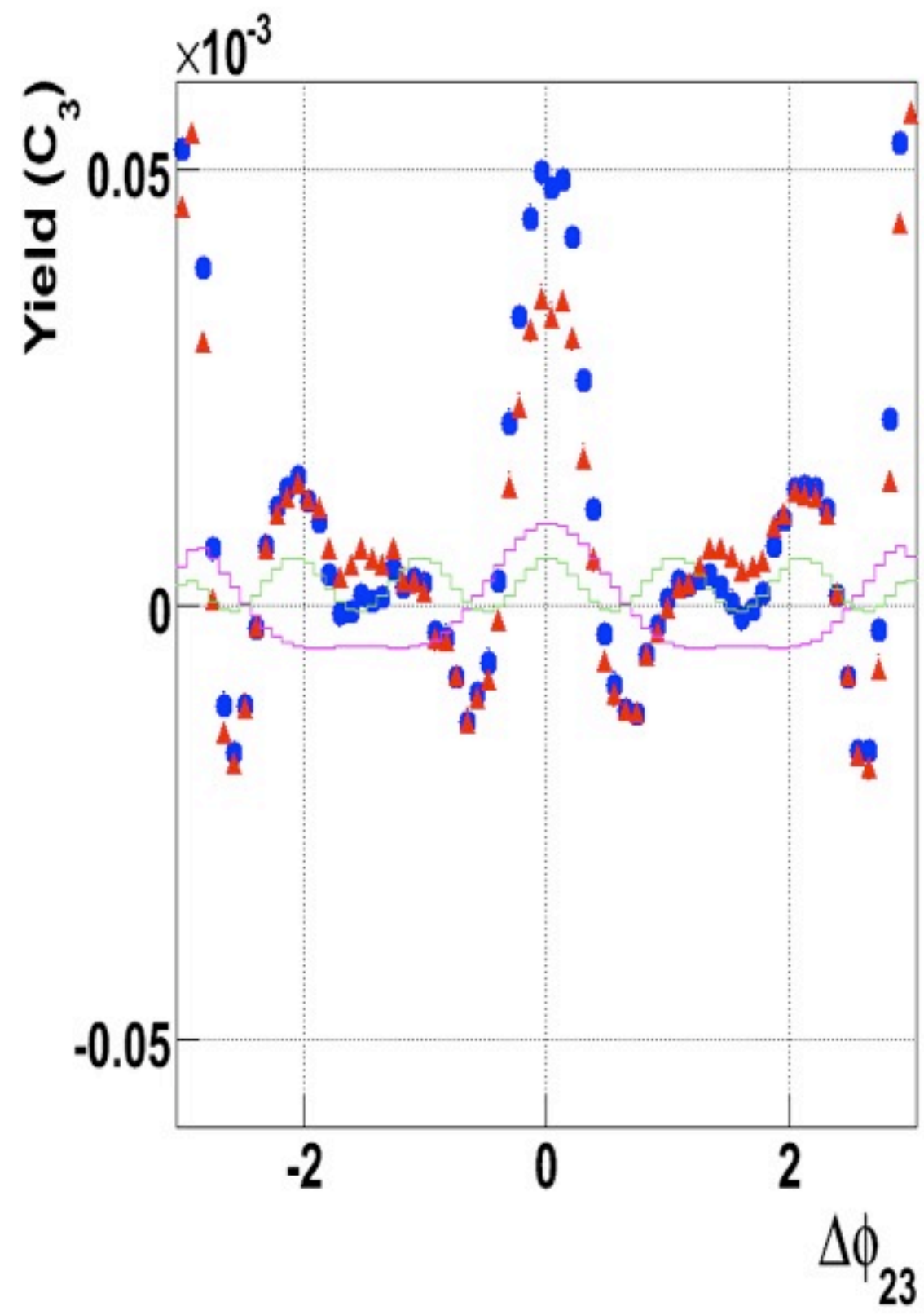
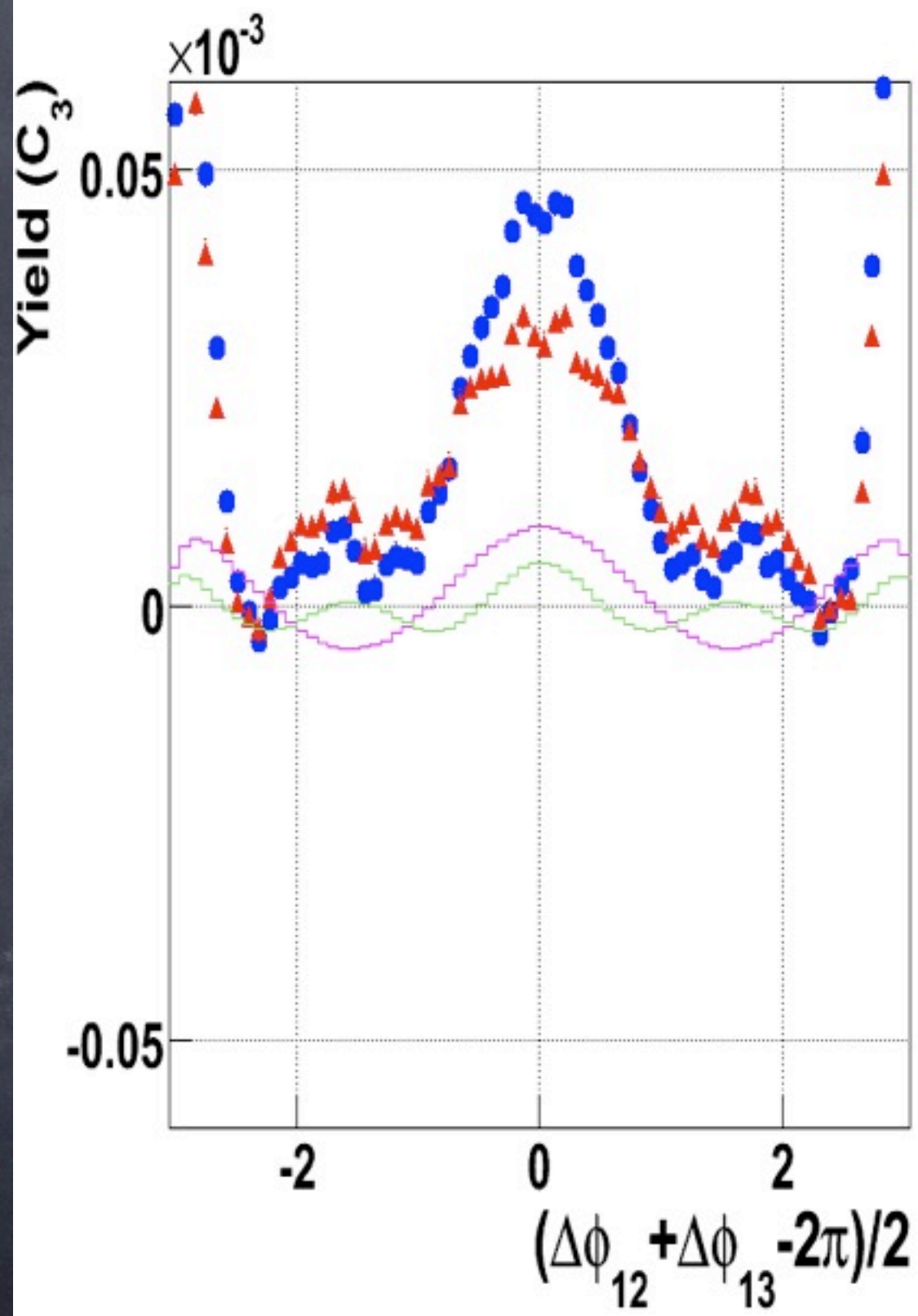
80–50%

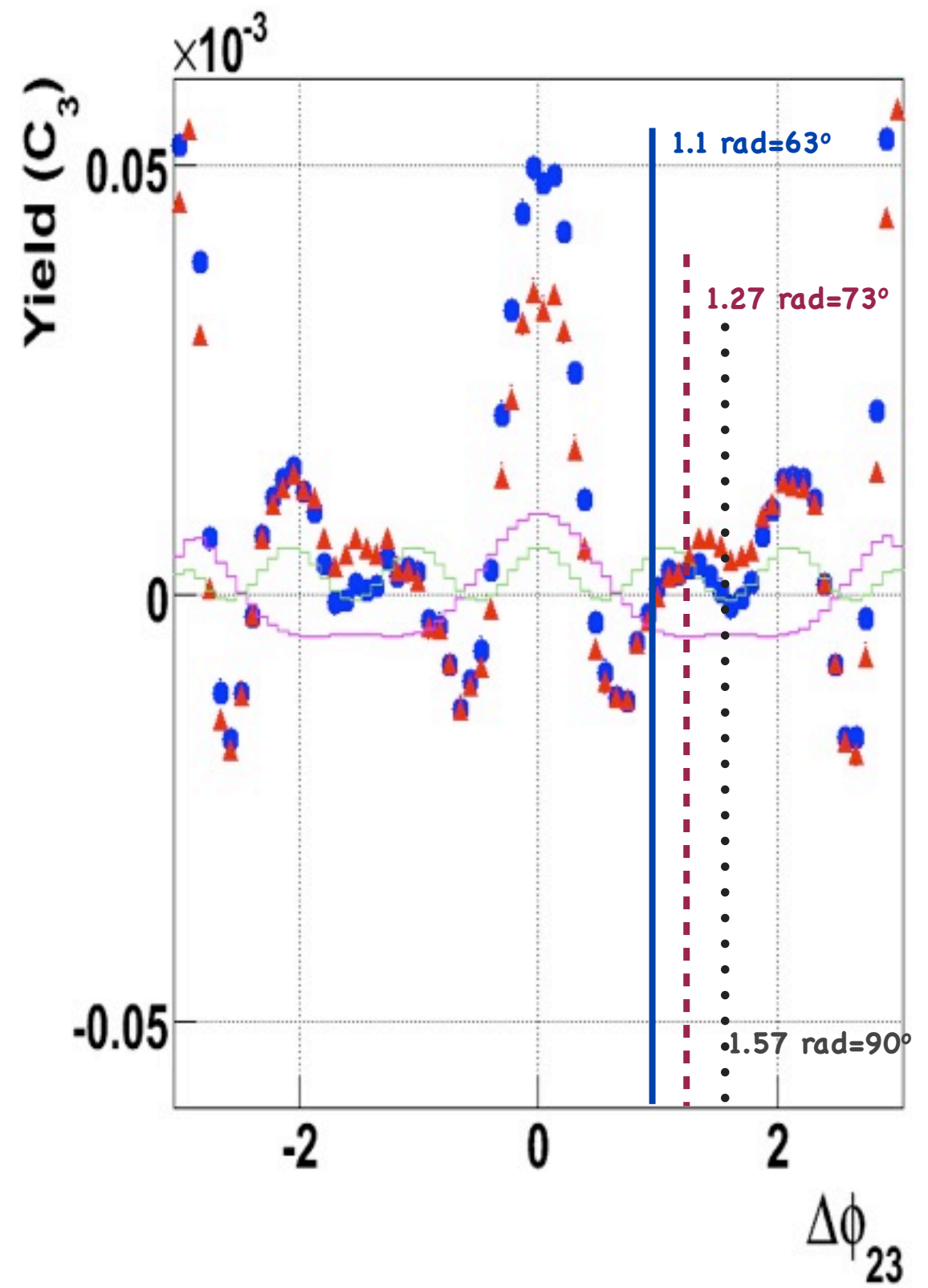
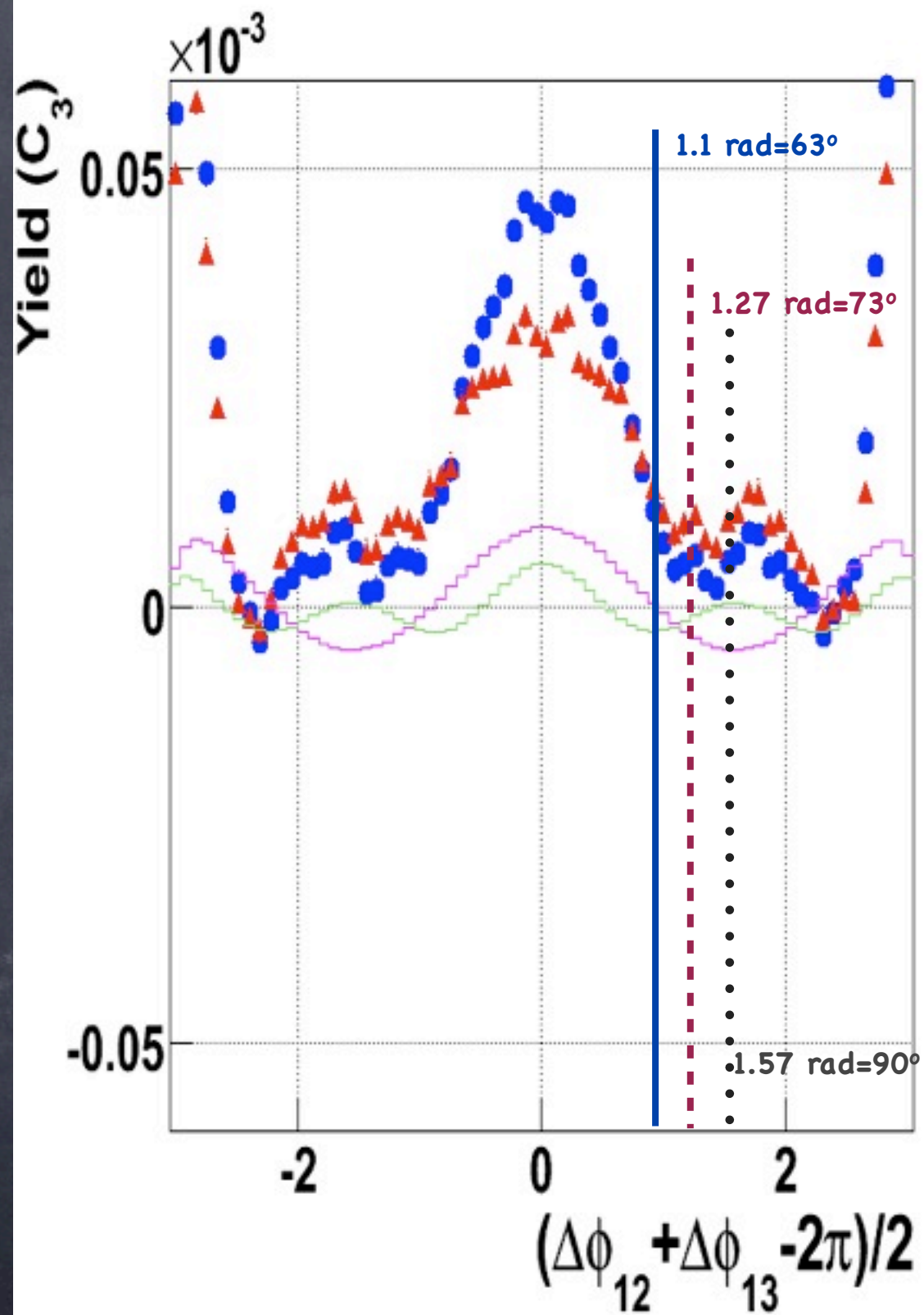
50–30%

30–10%

10–0%



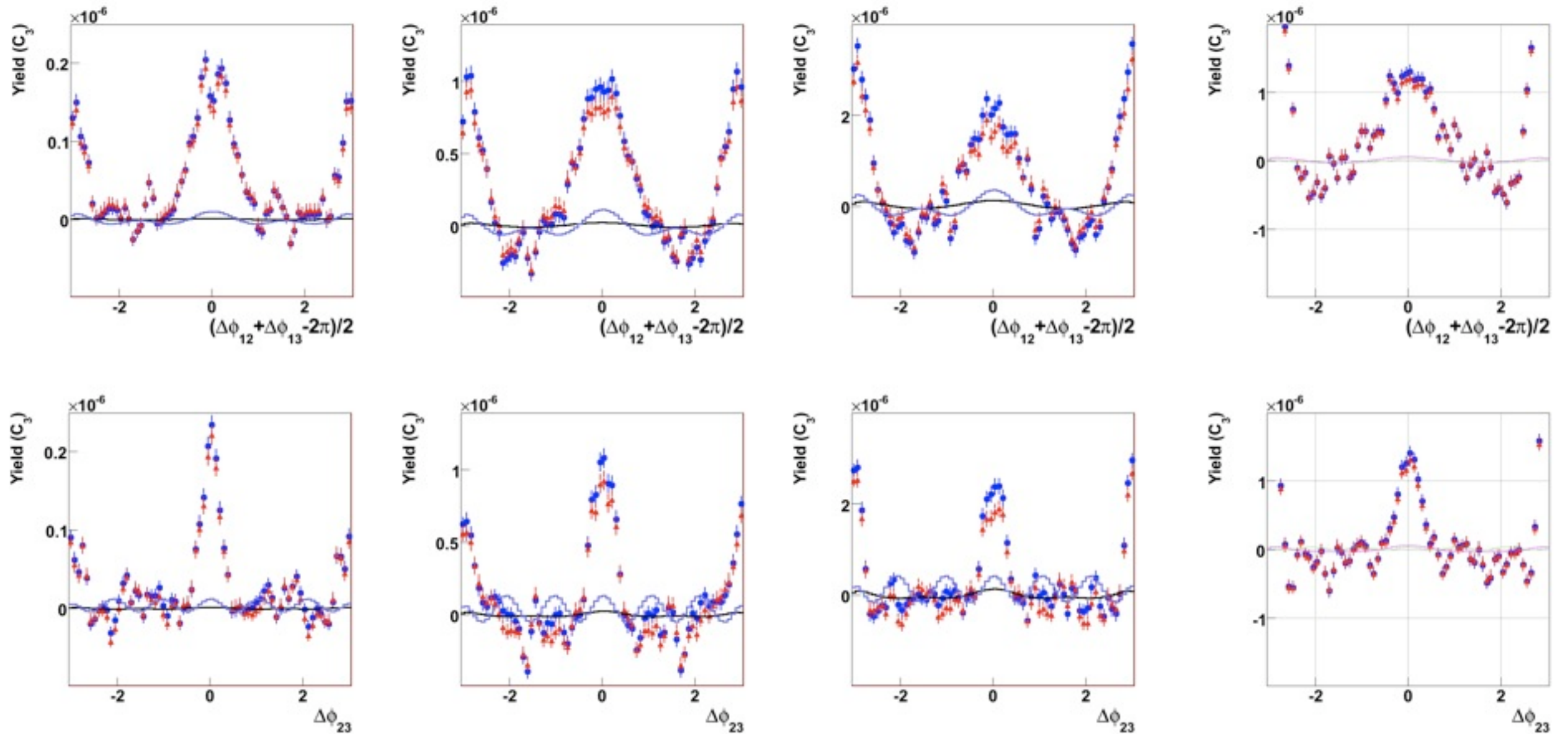




Summary

- ZYAM/Model Based Methods find indications of conical-like signal, BUT
- 3-Cumulant enables least biased search for conical emission.
- Sensitive to (approx) signal/background $> \sim 0.1 v_2^4$
- No signal found with cumulant method in Au+Au at 200 GeV.

Au+Au 200 GeV 1:3-20 GeV, 2&3: 2-3 GeV



80-50%

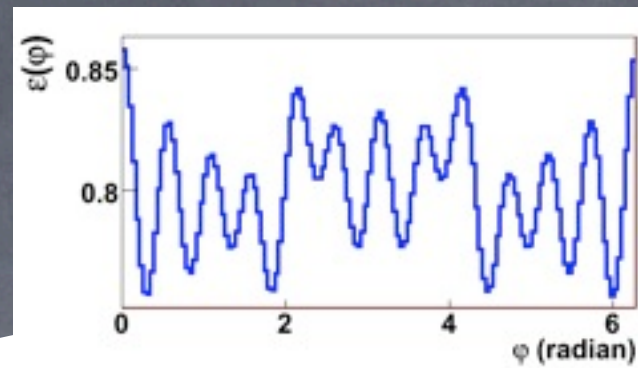
50-30%

30-10%

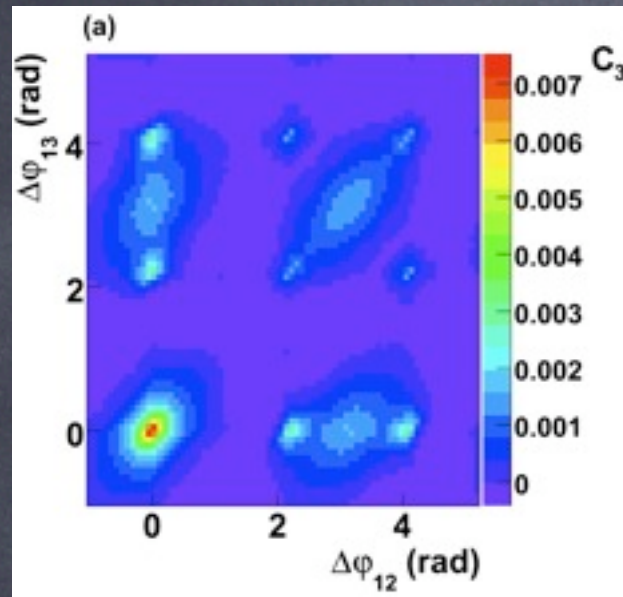
10-0%



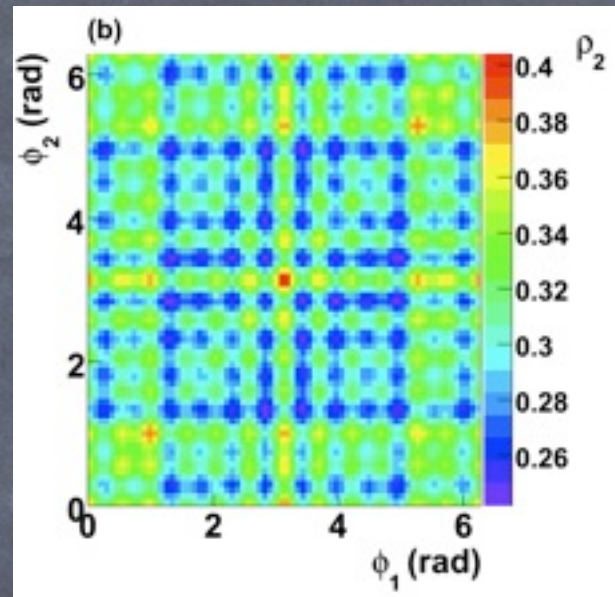
Experimental Robustness



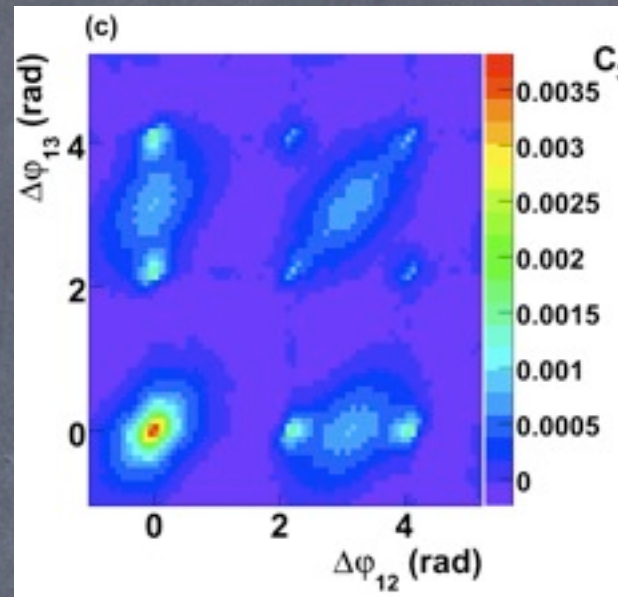
Detection
Efficiency



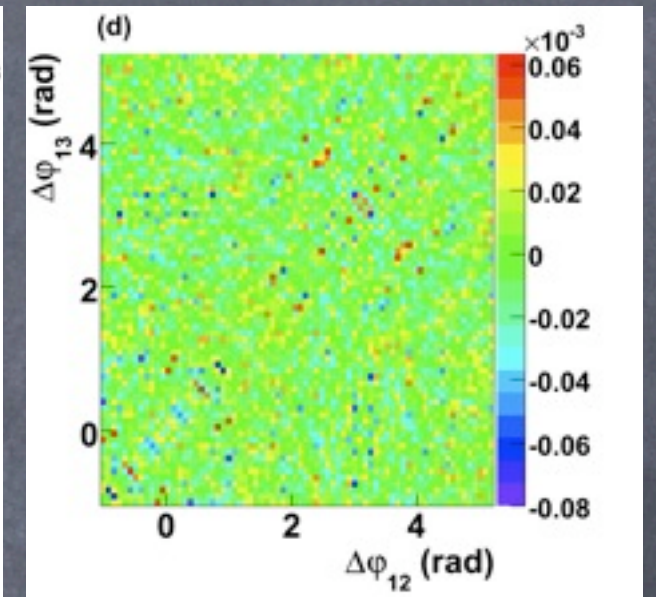
Simulated
3-Cumulant



2-particle
density



Measured
3-Cumulant

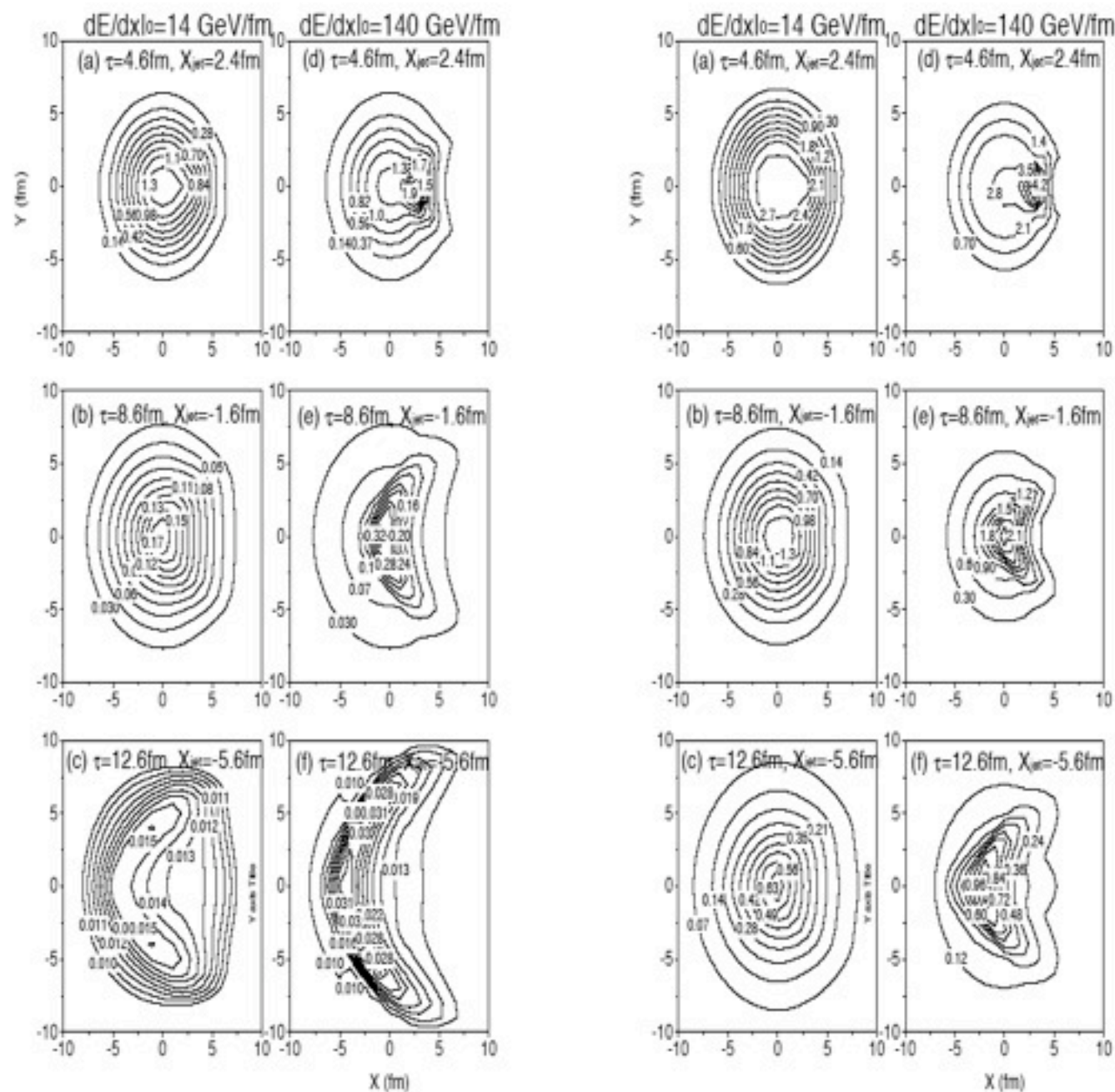


$(c) - 0.8^3(a)$

Measurement technique is robust!

Mach Cone – Hydro Calculation

A. K. Chaudhuri, U. Heinz, nucl-th/0503028



Fast Parton

$v_{\text{jet}}=0.2c$ Fast
Parton

